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AERONAUTICAL ENGINEERING

VOLUME ONE

R. TATHAM

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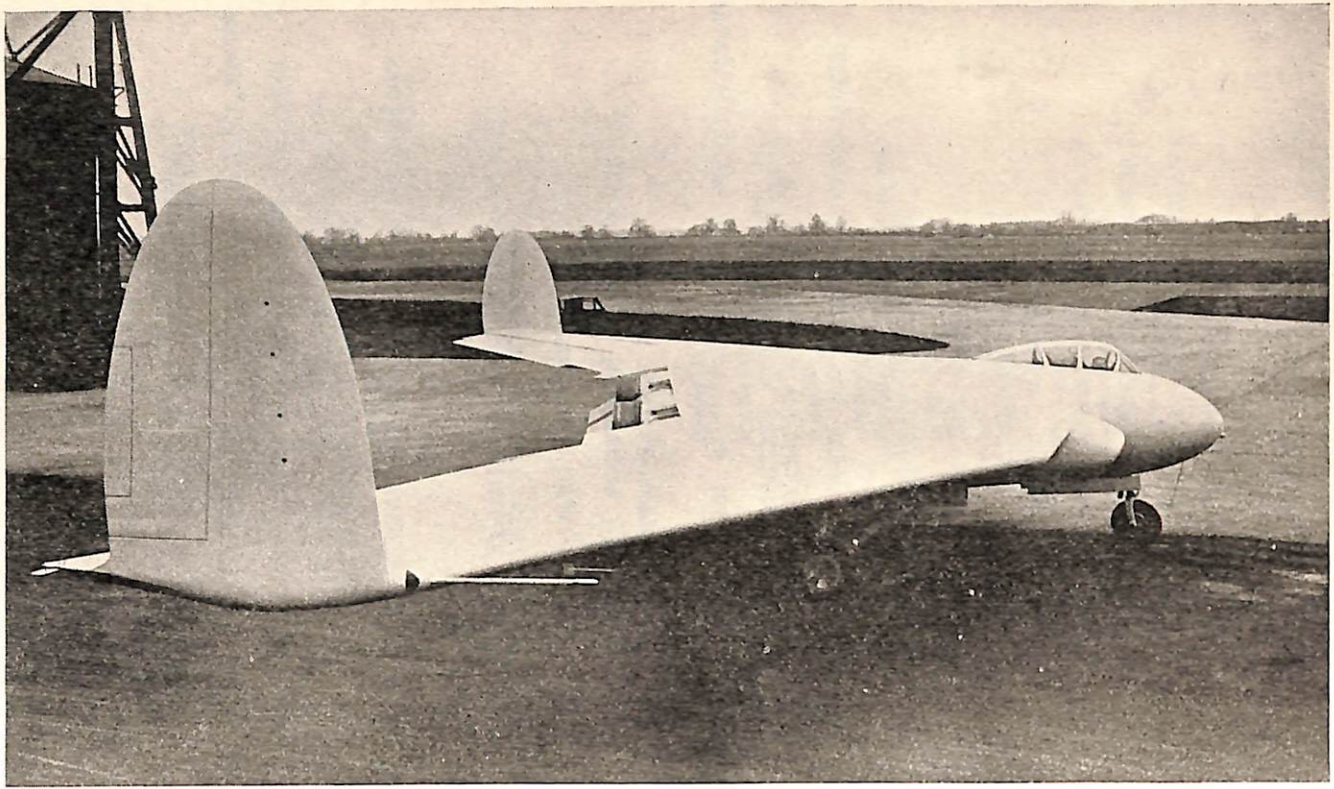

Frontispiece.

(By courtesy of Sir W. G. Armstrong Whitworth Aircraft Ltd.)

A.W.52. JET-PROPELLED TAILLESS AIRCRAFT.

THE TECHNICAL COLLEGE SERIES

AERONAUTICAL ENGINEERING

FOR
NATIONAL CERTIFICATE

BY

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VOLUME I

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GENERAL EDITOR'S FOREWORD

TECHNICAL education stands on the verge of a great advance, probably the greatest in its history. Throughout the country there is general recognition of the necessity for a great expansion of the facilities which are available for thorough and systematic training for science and industry. This desire finds expression in the provision which has been made in the new Education Act for all forms of technical education. During the war many thousands of men and women have been compelled to undergo brief and hastily improvised courses in scientific and technical processes, and this has helped to bring home to them, and to most people, the importance of systematic technical education in the conditions of modern life.

There is general recognition of the well-founded conclusions of experts that the economic prosperity of this country will, more than ever before, be dependent upon the efficiency and adaptability of our system of scientific and technical training.

It is the hope and ambition of those who have planned this series of technical books that it will meet the demands for new books which will follow these developments. New measures require new books, and it is hoped that this series will not only assist in meeting these requirements, but will incorporate those new methods and processes which have been introduced during the war years.

Fundamentally the books of the series are based, whenever this is necessary, on the Course system, which is now an established feature of technical education. Accordingly, in basic subjects, there will be suitable volumes for the successive years of the Course. It is recognised that there must be differences of opinion among teachers as to the sequence and contents of the work in the different years of a Course. But in general it is hoped that the treatment of the subjects will be sufficiently flexible to meet the needs of most teachers and students.

P. ABBOTT.

PREFACE

THE term " Aeronautical Engineering " covers a wide range of subjects, and it would be impossible in one book to attempt a full treatment. In the present instance it is hoped that this first volume contains sufficient information to enable the student to cover the major aeronautical subjects of Aerodynamics, and Aircraft Structures and Design, to the standard required for the S2 and S3 courses for the Ordinary National Certificate or the first year of a three years' full-time course for a Higher National Diploma. Although a certain amount of the theory of Strength of Materials has been included, since this is essential to a study of Aircraft Structures, no attempt has been made to present a comprehensive treatment of this and other subjects, such as Mechanics and Metallurgy, a knowledge of which is necessary for a complete understanding of Aeronautical Engineering. Many excellent books on these subjects exist, and the student should refer to such books for specialist information. No avoidance of elementary calculus has been attempted, the author being of the opinion that the over-simplification necessitated by omitting reference to what so many students regard as advanced mathematics tends ultimately to confuse the student and often requires him to accept too many statements without any proof. Following the usual syllabus for National Certificate the amount of Aerodynamics contained in this volume is small and of a very elementary nature. Subsequent volumes, however, will extend the major subjects to the standard required for the Higher National Certificate. For the convenience of the student the main principles of Mechanics required are given in an Appendix as a reference, and a small number of examples to be worked by the student has been added to most chapters.

Acknowledgments are due to several people for their kind assistance in the preparation of this book—to Miss M. A. Keeping, who typed the greater part of the manuscript; to my colleague, Mr. J. A. Choate, and my students, H. T. Howard and T. N. Corkhill, who prepared many of the diagrams; to my student, H. Rewitsch, who took the smoke tunnel photographs; and to Wing-Commander H. E. Falkner for valuable criticism and advice in the early stages

of the preparation. For permission to reproduce the photographs of aeroplanes, thanks are owed to *Flight*, Messrs. Percival Aircraft Ltd., Sir W. G. Armstrong Whitworth Aircraft Ltd., and to the Bristol Aeroplane Co. Ltd.

R. T.

LOUGHBOROUGH.

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LIST OF SYMBOLS USED IN VOL. I

A = Cross-sectional Area ; Aspect Ratio
C = Constant ; Coefficient
D = Drag ; Diameter
E = Young's Modulus
F = Force
G = Modulus of Rigidity
H = 1st Moment of Area
I = 2nd Moment of Area
$\mathfrak{J}$ = Polar Moment of Inertia ; Torsion Constant
K = Constant
L = Length ; Lift
M = Mass ; Moment
P = Force
R = Radius ; Resultant Force ; Reaction
S = Area (especially Wing Area)
T = Force (usually tensile) ; Temperature (Absolute) Torque
V = Volume ; Velocity
W = Weight
Z = Section Modulus
a = acceleration
b = wing span
c = chord ;
d = diameter
f = stress
g = acceleration due to gravity
h = height
k = constant
l = length
m = mass
p = stress ; pressure
q = shear stress ; dynamic pressure
r = radius
s = length of curve ; semi-span
t = temperature
u = velocity
v = velocity
w = wt./unit volume ; wing loading ; intensity of loading on beam
x = distance
y = distance
z = distance
α = angle of attack
β = angle
γ = angle
Δ, δ = small increment
ε = strain
η = efficiency
θ = angle
μ = coefficient of viscosity
ν = coefficient of kinematic viscosity
ρ = density
σ = relative density ; Poisson's ratio
ϕ = shear strain ; angle
ψ = angle
Ω, ω = angular velocity

Other symbols are defined as and when they occur.

CHAPTER I

THE AEROPLANE

1.1. Historical

In a book intended for students of Aeronautics, it is considered that a brief review of the work which led ultimately to successful powered flight by man will not be out of place.

From the earliest days man has been fired with the ambition to fly, and it is not unlikely that the myth of Dædalus and Icarus is merely the distorted story of a man who tried to imitate birds. Nearly all the early attempts at flight were based on the idea that man must provide himself with a substitute for wings, which could be worked by his own muscular strength.

Amongst early experimenters (A.D. 1065), an English monk, Oliver of Malmesbury, is stated to have made wings which he attached to his hands and feet, and to have crippled himself for life in attempting to use them. Later, about the year A.D. 1500, an Italian, Giovanni Danti, built a form of gliding apparatus with which he launched himself from the top of a tall tower in Perugia. After a flight of some 250 yards, he crashed on to a roof owing to the collapse of a wing main stay. It seems probable that Danti based his design on the work of Leonardo da Vinci, who contributed theoretical work of great value to the study of flight. Two others, who profited by da Vinci's work, were Guidotti and Verangio, the former building a glider with which he made several flights up to about 400 yards ending, like Danti, with a fall on to a roof. Verangio made parachutes to designs suggested by da Vinci, and experimented with them by jumping from high buildings. Other experimenters carried on until, in 1680, the belief that man could construct a machine which would enable him to fly using his own muscular strength as motive power was shattered by the publication of a book *De Motu Animalium* by J. A. Borelli, whose work followed much the same lines as da Vinci.

However, by the eighteenth century serious investigations were again proceeding, but when in 1783 the first hot-air balloon, invented by the brothers Montgolfier, ascended, attempts at flight by mechanical means once more ceased for a time. Not many years

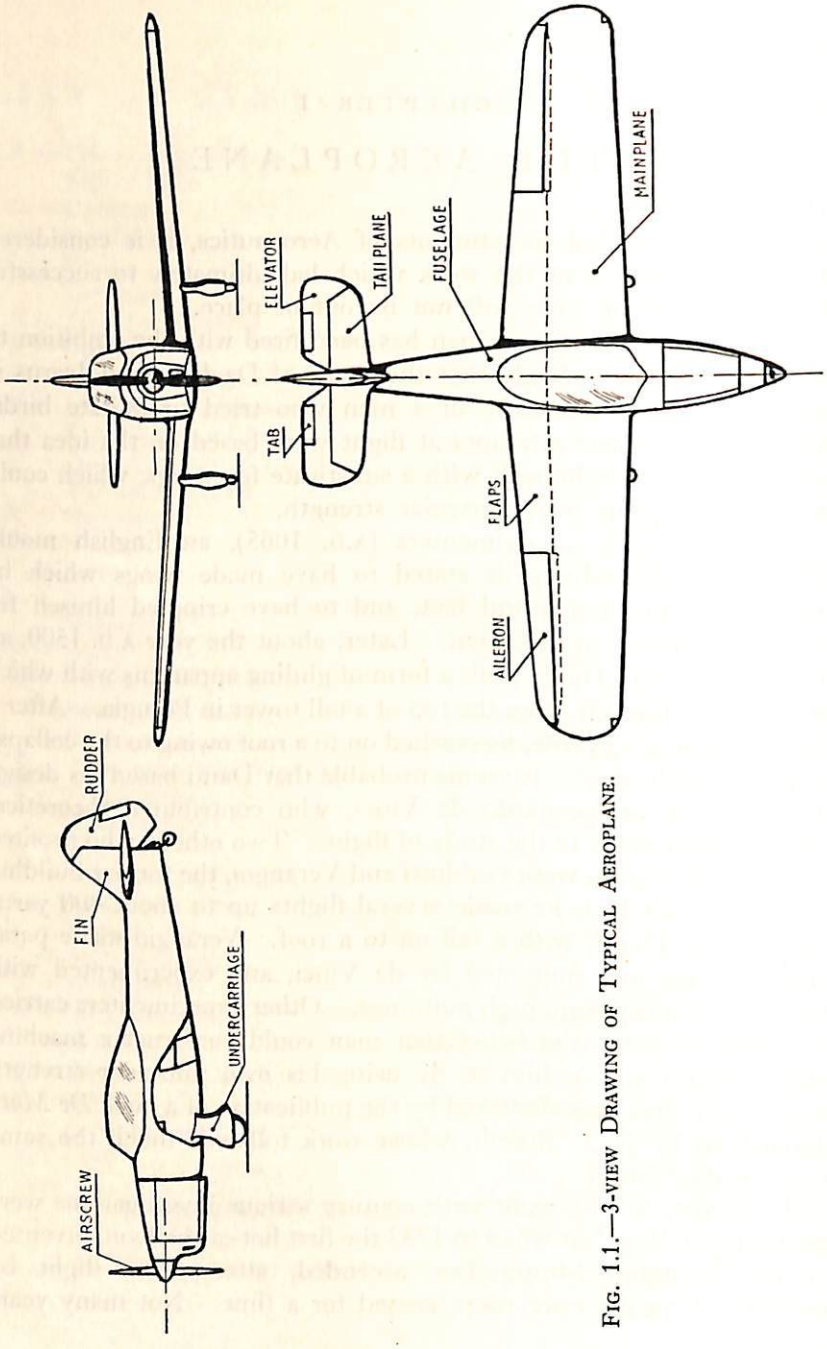


FIG. 1.1.—3-VIEW DRAWING OF TYPICAL AEROPLANE.

later Sir George Cayley published his theories on mechanical flight but, although he settled many problems in connection with powered flight, nearly 100 years elapsed before his suggestions could be practically realised.

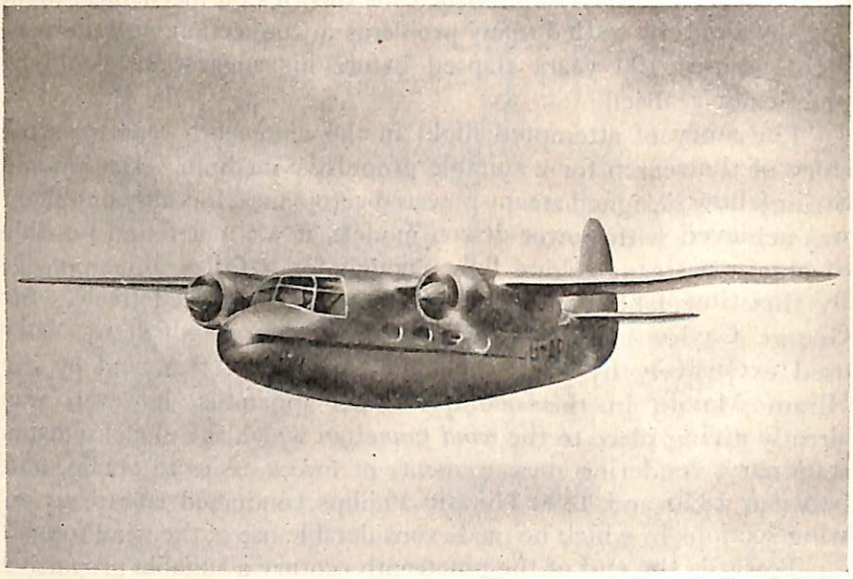
The story of attempted flight in the nineteenth century is the story of the search for a suitable propulsive medium. Henson and Stringfellow designed steam-powered aeroplanes, but although flight was achieved with power-driven models, it was not found possible to obtain a steam engine light enough for a full-scale aeroplane. By this time laboratory experiments were being used freely. Sir George Cayley had employed the *whirling arm*, which was later used extensively by Professor Langley in the U.S.A. and by Sir Hiram Maxim in this country. This apparatus, however, was already giving place to the *wind tunnel*, in which the model remains stationary, rendering measurements of forces easier to obtain, and between 1870 and 1890 Horatio Phillips conducted researches on wing sections in which he made considerable use of the wind tunnel.

Towards the end of the nineteenth century a suitable propulsive medium appeared in the form of the internal combustion engine. It was soon realised that one of the main problems of flight was stability and control, and attempts at finding a solution followed three lines. Lilienthal, Pilcher and Chanute, amongst others, made experiments with man-carrying gliders ; a theoretical approach was made by Dr. Lanchester ; and experiments on models in wind tunnels led to the successful flight on December 17th, 1903, by the Wright brothers.

Since that historic day, progress has been rapid, both in aerodynamic theory and in the practical development of the aeroplane. The successful application by Whittle of the gas turbine to jet propulsion has led to a state where the aeroplane lags behind the propulsive medium. The speed record now stands at well over 600 m.p.h., and with further developments in aircraft design, speeds greater than the speed of sound are quite likely to be achieved in the near future. However, still much research and development remains to be done, both to enable higher speeds to be reached, and to ensure greater safety in flight.

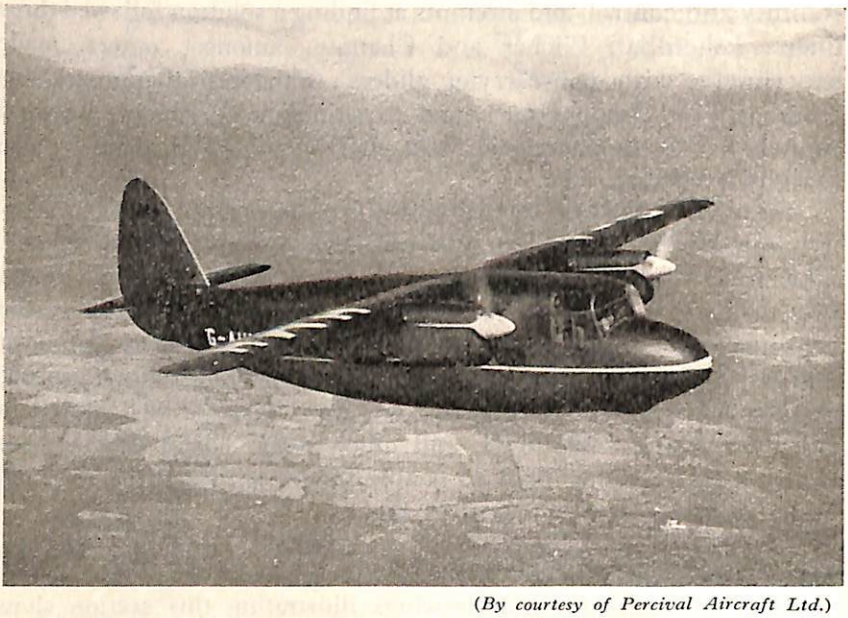
1.2. The Aeroplane To-day

The photographs and drawings illustrating this section show examples of modern aircraft. In particular Fig. 1.1 shows a low-



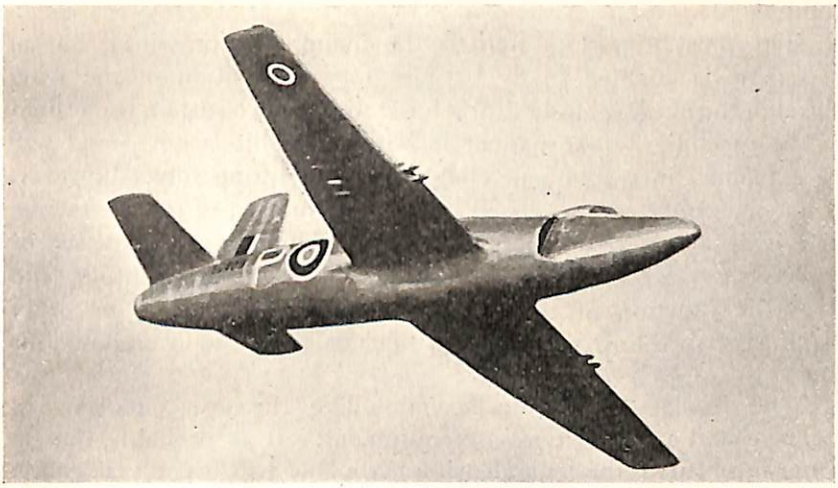
(By courtesy of Percival Aircraft Ltd.)

FIG. 1.2.—PERCIVAL "PRINCE," MEDIUM TRANSPORT, 8–10 PASSENGERS.



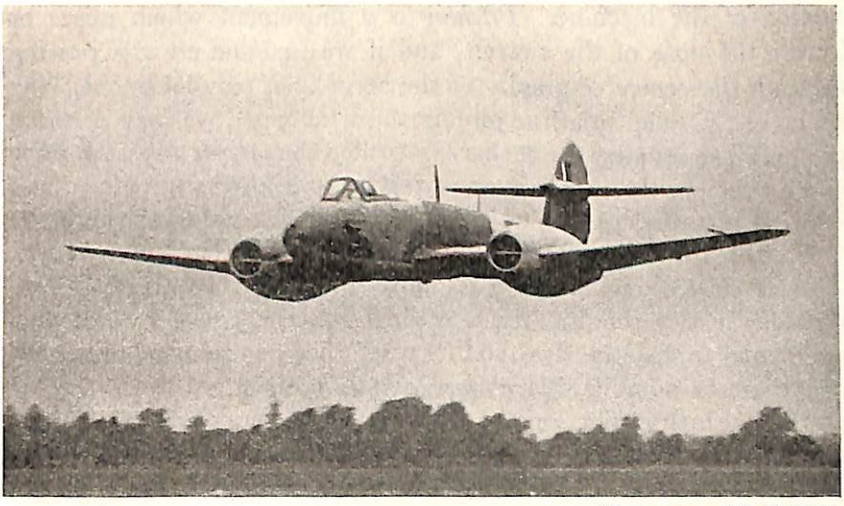
(By courtesy of Percival Aircraft Ltd.)

FIG. 1.3.—PERCIVAL "MERGANSER," PROTOTYPE LIGHT TRANSPORT, 5–8 PASSENGERS.



(*By courtesy of "Flight."*)

FIG. 1.4.—VICKERS ARMSTRONGS (SUPERMARINE) "ATTACKER."



(*By courtesy of "Flight."*)

FIG. 1.5.—GLOSTER "METEOR."

wing monoplane with its components named, and in the succeeding paragraphs a brief indication will be given of the function of these components.

The most important item is the mainplane (or wing), for an aeroplane is enabled to fly because forward motion of the wing through the air develops a lifting force sufficient to sustain the weight of the machine. The manner in which this lift is developed will be explained in subsequent chapters. The lifting force, however, is not the only force produced by forward motion of the aeroplane. The wing and every other part of the machine exposed to the air experience a force known as *drag*, which resists the motion, and it is the function of the engine-airscrew combination, or other propulsive medium, to provide sufficient thrust to overcome this drag.

The fuselage serves to accommodate the crew, passengers, baggage and various items of equipment. It is probable that at some future date the true all-wing aeroplane will become a reality, when the fuselage will no longer be required, and the whole of the load will be carried inside the wing itself.

In order to explain simply the functions of the control surfaces of an aeroplane we must define certain terms used to describe the motion of the machine. *Pitching* is a movement which raises or lowers the nose of the aircraft, and if we imagine an axis passing through the centre of gravity of the aeroplane, parallel to the wing span (i.e., parallel to a line joining the wing tips), we may describe pitching as a rotation about this axis (called the *lateral axis*). *Rolling* is a movement which raises one wing tip and lowers the other, and may be regarded as a rotation about an axis passing through the C.G. in a fore and aft direction (known as the *longitudinal axis*). *Yawing* causes one wing tip to move ahead of the other, and we consider it as a rotation about a third axis through the C.G. perpendicular to the other two, so that when the wing span is horizontal, i.e., there is no roll, this *normal axis* is vertical.

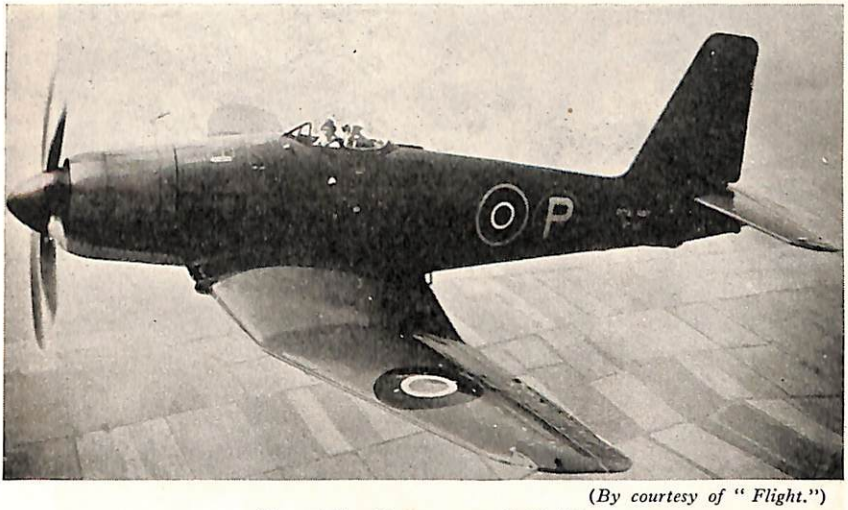
The primary functions of the control surfaces are to produce the required rotations about these three axes, as indicated below.

The tailplane and fin are both required to give stability, the former in pitch and the latter in yaw.

The main control surfaces are the ailerons, elevators and rudder. The ailerons are hinged portions at the extremities of the wing, and are so connected that when one aileron moves down the other moves

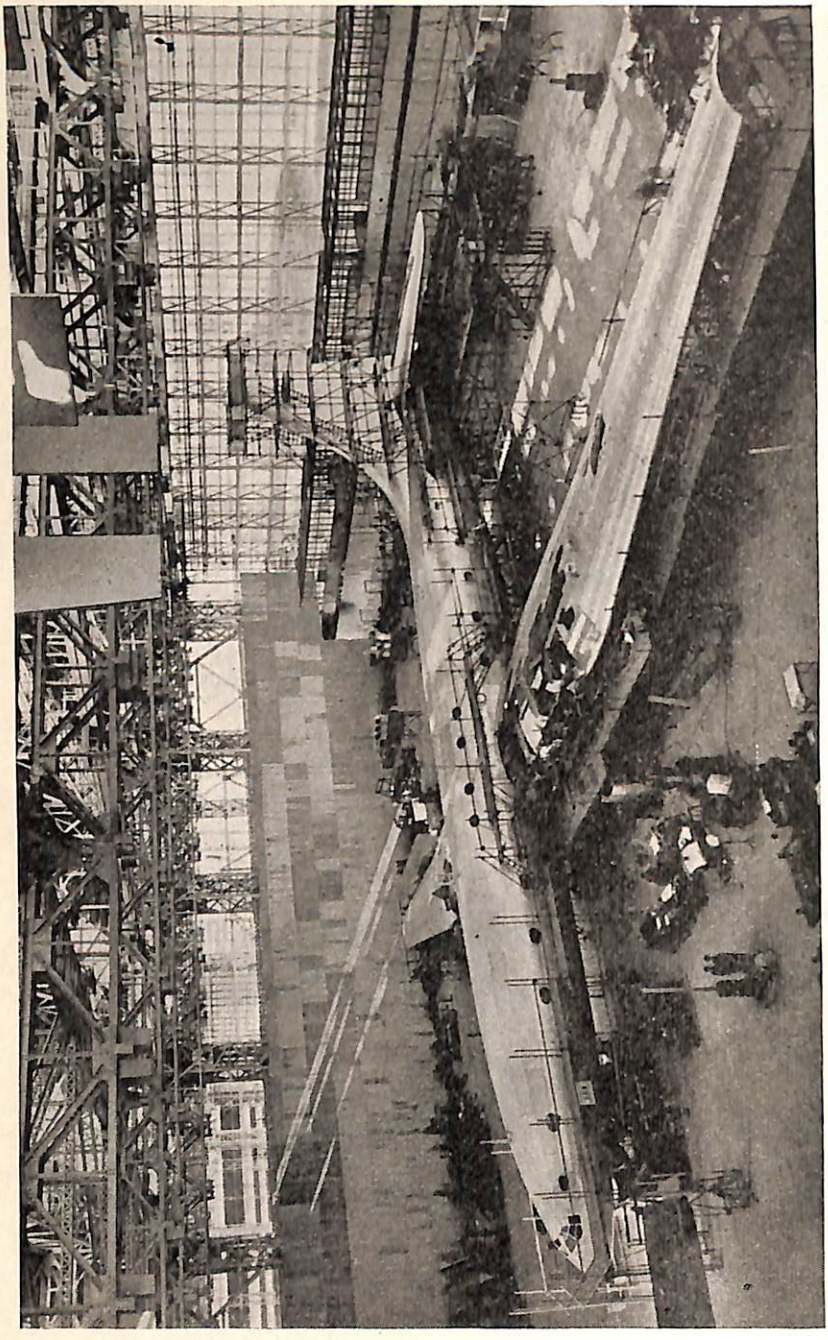
(By courtesy of Percival Aircraft Ltd.)

FIG. 1.6.—PERCIVAL " PRENTICE " TRAINING AIRCRAFT.



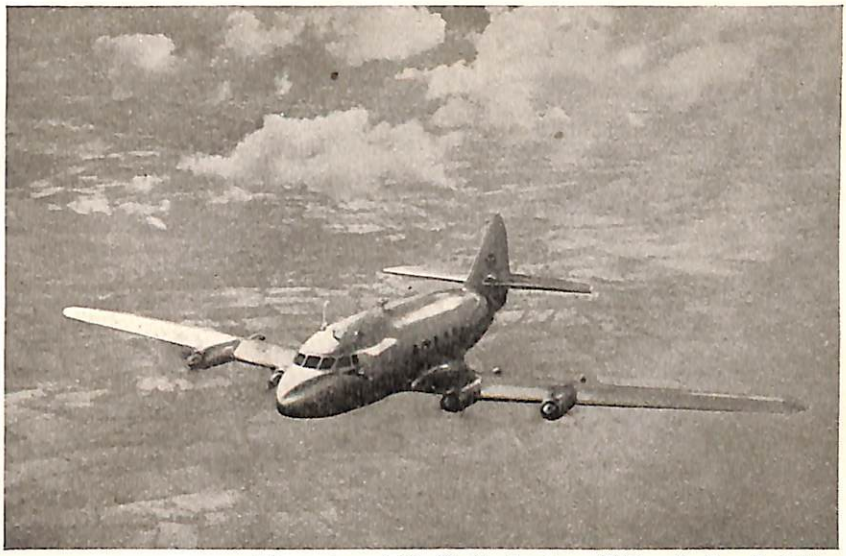
(By courtesy of " Flight.")

FIG. 1.7.—BLACKBURN S.28/43.



(*By courtesy of Bristol Aeroplane Co. Ltd.*)

FIG. 1.8.—BRISTOL "BRABAZON" UNDER CONSTRUCTION.



(By courtesy of Sir W. G. Armstrong Whitworth Aircraft Ltd.)

FIG. 1.9.—ARMSTRONG WHITWORTH "APOLLO."

up. In general, downwards movement of an aileron causes an increase of lift on the portion of the wing affected by it, and upward movement produces a decrease of lift. Consequently, a couple is produced which causes the aeroplane to bank or roll. The elevators, hinged to the tailplane, are used to raise or lower the nose of the machine, and the rudder gives directional control, causing the aeroplane to yaw to one side or the other. As will be explained in more detail later, these controls are not completely independent—use of one control surface will often require the use of another simultaneously. The flaps, extending usually over the portions of the wing inboard of the ailerons, are used primarily to steepen the angle of approach in a glide and to enable the aeroplane to be landed at a reasonably low speed.

The undercarriage is a necessary evil, which is required so that the aeroplane can be taxied on the ground, but which in the air provides a lot of extra drag and weight. In many cases the undercarriage may be retracted in flight so that the unwanted drag is eliminated, but the weight remains, and part of the lift developed by the wing is used in carrying the undercarriage.

(*By courtesy of Bristol Aeroplane Co. Ltd.*)

FIG. 1.10.—BRISTOL HELICOPTER TYPE 171.

(*By courtesy of "Flight."*)

FIG. 1.11.—DE HAVILLAND "TIGER MOTH" BIPLANE TRAINING AIRCRAFT.

Although the above remarks have been made with reference to the aeroplane of Fig. 1.1, they apply generally to all the other aeroplanes shown. Types of aeroplanes other than the low-wing monoplane may be observed in the photographs of this chapter.

1.3. The Autogyro and Helicopter

Both the autogyro and the helicopter differ from the conventional aeroplane in that the lift is derived from a *Rotor* instead of a fixed wing. The essential difference between the two is that the rotor of the helicopter is driven and, by suitable control, is used to provide thrust for forward motion as well as lift, whilst the rotor of the autogyro rotates freely as a result of forward motion of the machine, a separate propeller being installed to provide the thrust as on a conventional aeroplane.

To-day the helicopter is an accomplished fact, although a large amount of research and development is required to enable it to compete economically with the conventional aeroplane in the commercial field.

CHAPTER II

PROPERTIES OF THE ATMOSPHERE

2.1

It is obvious that, if we are to consider the conditions that arise when an aeroplane flies, we must understand something of the properties of the medium in which flight takes place ; it is, indeed, necessary to investigate the properties of air at rest before we can talk of what happens when a body moves through air or, what amounts to the same thing, when air moves past a body, such as an aeroplane wing. This chapter is devoted to the consideration of the pressure, density and temperature of air at rest, and of the variation of these quantities with each other and with altitude.

2.2. Pressure

The pressure of the air at rest arises from the weight of the atmosphere above the point at which the pressure is measured. At sea level the atmospheric pressure is approximately 2116 lb. per sq. ft., that is to say, the air presses down on each square foot with a force of approximately 2116 lb. That this pressure does not make itself apparent as a force tending to thrust this book from the reader's hands is due to the fact that the pressure at any given point in a fluid or a gas, such as air, acts equally in all directions, so that there is an upthrust on the book of equal value to the downward force.

The pressure decreases with increase in altitude, and for a small difference in height, h, between two points A and B, A being lower than B, the difference in pressure is given by

$$p_A - p_B = wh \quad . \quad . \quad . \quad . \quad \text{Eq. 2.1}$$

where w is the local weight per unit volume of the air, p_A and p_B being the atmospheric pressure at A and B, respectively. This equation is true for air only if h is small, since air is compressible, so that the weight per unit volume is greater at low levels than at high levels—i.e., w varies with height, but for small differences in height it may be assumed constant.

Two other properties of pressure, which we shall find useful

when we consider the measurement of pressure, are (1) that the pressure in a fluid, or gas, at rest is the same at all points in the same horizontal plane, and (2) that a pressure applied to any point of an incompressible fluid is transmitted equally to all parts of that fluid.

2.3. Measurement of Pressure—Pressure Head

It is common practice to speak of a pressure as " so many inches of mercury, or water, or other fluid " instead of " so many pounds per square foot." In these cases it is the *pressure head* which is actually quoted, this being the height of a column of liquid which can be supported by a given pressure. For example, instead of saying that the atmospheric pressure at sea level is 2116 lb. per sq. ft., we may refer to it as 29·92 in. of mercury, or as 760 mm. of mercury, by quoting the pressure head instead of the pressure.

Measurement of pressure is generally effected by measuring the corresponding pressure head and then, if required, the head may be converted into units of pressure.

2.4. Types of Pressure Gauge

The simplest type of gauge for measuring a single pressure is shown in Fig. 2.1. It consists of a tube, closed at one end, exhausted of air, placed vertically, open end downwards, in a vessel containing some liquid, say mercury. The pressure to be measured is then applied to the free surface of the mercury. For simplicity, let us suppose that this gauge is being used to measure the atmospheric pressure, the free surface of the mercury being exposed to atmosphere. Mercury will rise in the tube to a height h, when the weight of the mercury of height h is exactly equal to the upward thrust on the base of this column, which will be due to the atmospheric pressure on the free surface; h is then the pressure head. If h be measured in feet, and the weight per unit volume of the mercury be w lb. per cu. ft., and if the cross-sectional area of the tube be a sq. ft., the atmospheric pressure p_0 will be given by :—

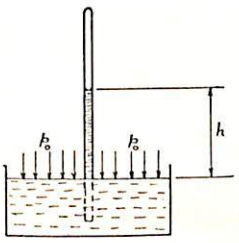


FIG. 2.1.—SIMPLE PRESSURE GAUGE.

$$p_0 \times a = w \times h \times a$$

i.e. $$p_0 = wh \text{ lb. per sq. ft.} \quad . \quad . \qquad \text{Eq. 2.2}$$

This is a general equation relating any pressure and the corresponding pressure head. It should be noted that the cross-sectional area of the tube, a, does not appear in Equation 2.2, so that the pressure head is independent of the size of tube used. It is necessary only to measure the *height* of the column of liquid.

In Aerodynamics we are usually concerned with the measurement of the difference between two pressures—often a very small difference requiring accurate gauges. The basic form of a gauge for measuring a pressure difference is the simple U-tube manometer (Fig. 2.2 (a)). A pressure p_A is applied to one limb of the tube and a pressure p_B, assumed less than p_A, is applied to the other limb.

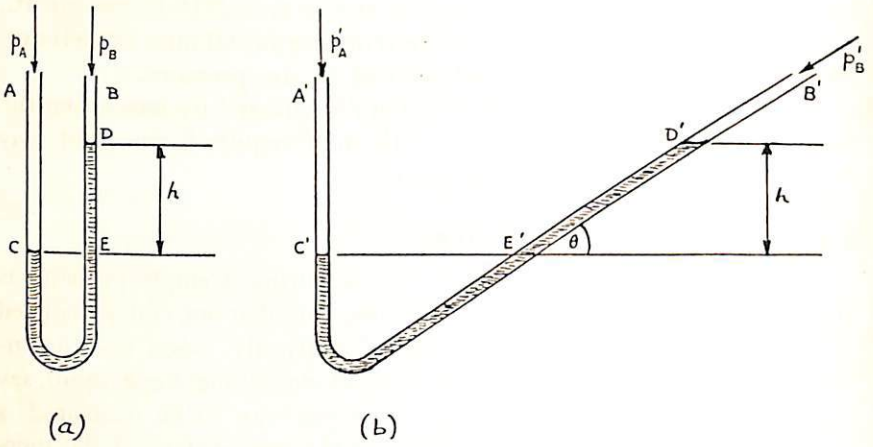


FIG. 2.2.—U-TUBE MANOMETER.

The liquid contained in the U-tube will stand at a higher level in limb B than in limb A, the position of the liquid being indicated by the portion CED. Since E is on the same horizontal level as C, the pressure at E will be equal to that at C, and hence to p_A. The pressure at D is p_B, and from Equation 2.1 it follows that

$$p_A = p_E = p_B + w\mathrm{DE}$$

where w is the weight per unit volume of the liquid.

Therefore, $$p_A - p_B = w\mathrm{DE} \quad . \quad . \quad . \quad \text{Eq. 2.3}$$

and DE is the pressure head corresponding to the difference between the pressures p_A and p_B.

This type of manometer is not accurate for small pressure differ-

ences owing to the difficulty in measuring the very small head involved. A magnification of the movement of the liquid can be obtained if limb B is sloping instead of vertical, as shown in Fig. 2.2 (b). The pressure head is still given by the vertical difference in height, and, from the figure

$$D'E' = \frac{DE}{\sin \theta}$$

which is greater than DE.

A further refinement is to make limb A of much larger bore than limb B. In this case a small, and often negligible, displacement of the liquid in A will cause a large displacement in B, rendering reading easier.

Equation 2.3 suggests the possibility of using liquids of different specific weights to cover several ranges of pressure differences. For example, to measure a pressure difference equal to atmospheric pressure (2116 lb. per sq. ft.) would require measurement of a head of water of $\frac{2116}{62{\cdot}5} = 33{\cdot}75$ ft. If, however, the liquid used in the gauge is mercury, of specific gravity 13·5, the head required is reduced to $\frac{2116}{62{\cdot}5 \times 13{\cdot}5} = 2{\cdot}493$ ft. $= 29{\cdot}92$ in.

A very accurate type of gauge, for use with very small differences of pressure, is the Chattock Gauge or Tilting Manometer, a diagrammatic sketch of which is shown in Fig. 2.3, with typical dimensions.

The gauge acts as a U-tube, the essential feature being that the surface of separation of two liquids is observed by a microscope. This surface of separation is maintained in a fixed position by tilting the gauge, the amount of tilt required being a measure of the difference between the pressures applied to the main vessels A and B. The lower portion of all three vessels A, B and C contain water or a saline solution, whilst the upper portion of the central vessel is filled with a liquid such as castor oil or Nujol. The level of the gauge is adjusted so that the surface of separation between the two liquids at the extremity of the small central tube is readily seen in the microscope. The frame carrying the glasswork is pivoted at D, and supported at E so that by rotation of the screwed support at E, by means of the handwheel, the vessel B can be raised relative to the vessel A, thus increasing the effective pressure head in B. The

handwheel is graduated so that the distance by which the point E is raised or lowered can be ascertained. To obtain the distance through which B is raised or lowered relative to A, this reading must be multiplied by the ratio of the length AB to the length DE; this ratio is known as the *gauge constant.*

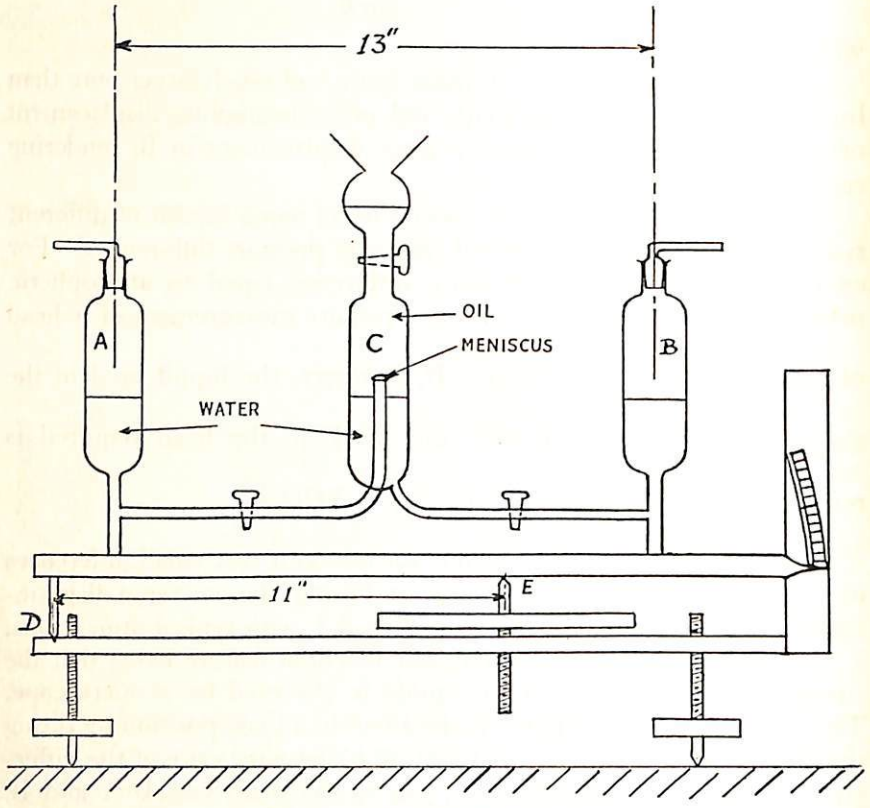


FIG. 2.3.—CHATTOCK GAUGE.

Having raised B by a certain amount to maintain the surface of separation in its original position, the difference between p_A and p_B is equal to the amount by which the pressure in vessel B has been increased due to raising the level of the water in it above that of the water in A.

Hence, if B is raised x in. relative to A, and if the weight per unit volume of the liquid in A and B is w lb. per cu. ft., the pressure

difference is given by

$$p_A - p_B = w\frac{x}{12} \text{ lb. per sq. ft.} \quad . \quad . \quad \text{Eq. 2.4}$$

The scale indicated on the right of the diagram is for convenience in counting complete revolutions of the handwheel. With a screw of 20 threads per in., and the handwheel graduated in 50 equal divisions, a movement of one division of the handwheel represents a movement of E through 0·001 in., a complete revolution of the handwheel corresponds to 0·05 in.

E.g., A Chattock gauge filled with liquid of specific gravity 1·06 is used to measure the difference between two pressures. The pitch of the screw thread of the gauge is $\frac{1}{20}$ in., the distance between the pivots is 9·95 in. and the distance between the centres of area of the two cups is 13·05 in. If 8·6 turns of the screw are required for balance, what is the pressure difference ?

Referring to Fig. 2.3, the distance CE is 9·95 in.; AB = 13·05 in.

$$x = 8{\cdot}6 \times \tfrac{1}{20} \times \frac{13{\cdot}05}{9{\cdot}95} \text{ in.} = 0{\cdot}563 \text{ in.}$$

The weight of water = 62·5 lb. per cu. ft., and specific gravity of the liquid in the gauge is 1·06, so that

$$w = 1{\cdot}06 \times 62{\cdot}5 \text{ lb. per cu. ft.}$$

$$\therefore p_A - p_B = \frac{w \text{ lb. per cu. ft.} \times x \text{ in.}}{12}$$

$$= \frac{1{\cdot}06 \times 62{\cdot}5 \times 0{\cdot}563}{12} \text{ lb. per sq. ft.}$$

$$= \mathbf{3{\cdot}11 \text{ lb. per sq. ft.}}$$

2.5. Weight, Mass and Density

Confusion often arises regarding the distinction between a pound weight and a pound mass, and in this section it is hoped to make this distinction clear. When we say that an article has a weight of 1 lb., we really mean that the force exerted on it by gravity is equal to the force which will produce in a mass of 1 lb. an acceleration of approximately 32·2 ft. per sec.^{2}. This acceleration varies from place to place, and we shall refer to it as g, understanding that the numerical value to be used is the value of the acceleration due

to gravity at whatever place we happen to be considering. Since force is equal to mass times acceleration, we may now unite :—

A force of 1 lb. weight = 1 lb. mass $\times$ g ft. per sec.^{2} Eq. 2.5

It is often desirable to choose our units of force and mass so that we can say : Unit force produces on unit mass unit acceleration. Obviously, from equation 2.5 we cannot do this if we use the pound as the unit of both force and mass ; consequently, we must use a different unit for one of these. In order to avoid confusion between the number g and the acceleration g ft. per sec.^{2}, let us use 32·2 as the numerical value of g in equation 2·5.

Then, 1 lb. wt. = 1 lb. mass $\times$ 32·2 ft. per sec.^{2} Eq. 2.6

This equation may be rewritten in two ways :—

$$\frac{1 \text{ lb. wt.}}{32{\cdot}2} = 1 \text{ lb. mass} \times 1 \text{ ft. per sec.}^2 \quad \text{Eq. 2.7}$$

or 1 lb. wt. = 32·2 lb. mass $\times$ 1 ft. per sec.^{2} Eq. 2.8

We will consider each of these equations separately. Equation 2.7 indicates how we may change our unit of force by using " 1 lb. wt. $\div$ 32·2 " as the unit instead of 1 lb. wt. The name given to this unit of force is the *poundal*, and using this definition equation 2·7 becomes

A force of 1 poundal = 1 lb. mass $\times$ 1 ft. per sec.^{2} Eq. 2.9

This system is commonly used by mathematicians, but often leads to confusion because in our everyday language we are accustomed to think of force in terms of pound weight. Equation 2·8 leads us to a system which fits in more easily with everyday usage, and which is the one used in aerodynamics. In this case we use 32·2 lb. instead of 1 lb. for our unit of mass, and we call this unit 1 slug.

Hence,

A force of 1 lb. wt. = A mass of 1 slug $\times$ 1 ft. per sec.^{2} Eq. 2.10

Using this system we can now discard the terms weight and mass in equation 2.10, and, remembering that the pound is now the unit of force, we have

A force of 1 lb. = 1 slug $\times$ 1 ft. per sec.^{2} Eq. 2.11

Since a mass of 1 lb. has a weight of 1 lb. (equation 2·5), it follows that a mass of 1 slug weighs 32·2 lb. ; i.e., we may define the slug as the mass of a body which has a weight of 32·2 lb.

When we say, for example, that air weighs 0·0765 lb. per cu. ft., we mean that air has a mass of 0·0765 lb. per cu. ft. The name given to mass per unit volume is *density*. Using the slug as the unit of mass we calculate that the density of air is $\frac{0{\cdot}0765}{32{\cdot}2} = 0{\cdot}00238$ slugs per cu. ft.

If a volume of 10 cu. ft. of air is given an acceleration of 50 ft. per sec.^{2}, the force required to produce the acceleration is given by :—

$$\text{Force (lb.)} = (10 \times 0{\cdot}00238) \times 50 = \mathbf{1{\cdot}190\ lb.}$$

since Force (lb.) = Mass (slugs) × Acceleration (ft. per sec.^{2}).

To find the weight of 10 cu. ft. of air we must determine the force required to produce an acceleration of 32·2 ft. per sec.^{2}.

Hence, weight (lb.) $= (10 \times 0{\cdot}00238) \times 32{\cdot}2 = \mathbf{0{\cdot}765\ lb.}$

For convenience, the above is summarised as follows :—

A force of 1 lb. produces on a mass of 1 lb. an acceleration of g ft. per sec.^{2}.

A force of 1 poundal produces on a mass of 1 lb. an acceleration of 1 ft. per sec.^{2}.

A force of 1 lb. produces on a mass of 1 slug an acceleration of 1 ft. per sec.^{2}.

$$1 \text{ slug} = g \text{ lb. mass}$$
$$g \text{ poundals} = 1 \text{ lb. wt.}$$

Except when otherwise stated, in this book density is expressed in slugs per cubic foot, and the following notation will be adopted :—

$W \equiv$ Weight
ρ (rho) $\equiv$ Density
$g \equiv$ Acceleration due to gravity
$a \equiv$ Acceleration in general
P or R or $F \equiv$ Force
$V \equiv$ Volume

Hence, in symbols, the equation connecting force, mass and acceleration, becomes

$$P \text{ (lb.)} = \rho \text{ (slugs per cu. ft.)} \times V \text{ (cu. ft.)} \times a \text{ (ft. per sec.}^2\text{)}$$
and $$W \text{ (lb.)} = \rho \text{ (slugs per cu. ft.)} \times V \text{ (cu. ft.)} \times g \text{ (ft. per sec.}^2\text{)}$$

2.6. Density of Air

The density of air, which is a gas, varies with changes in pressure and temperature, the relationship between the three quantities being the Equation of State, derived from a combination of Boyle's Law and Charles' Law for a perfect gas.

These two laws state :—

(1) For a given mass of a perfect gas, the product of the pressure and volume remains constant if the temperature is constant.

i.e., $pV = \text{constant}$ when T is constant . Eq. 2.12

(2) For a given mass of a perfect gas, the pressure is proportional to the absolute temperature if the volume remains constant.

i.e., $\frac{p}{T} = \text{constant}$ when V is constant . Eq. 2.13

The Equation of State is obtained from equation 2.12 and equation 2.13 as :—

$pV = RT$ where R is a constant . . Eq. 2.14

Density is mass per unit volume, so that if we consider V to be the volume of 1 lb. of the gas

$$\rho = \frac{1}{gV} \text{ slugs per cu. ft.}$$

and $$V = \frac{1}{g\rho}$$

Making this substitution in equation 2.14 gives :—

$$\frac{p}{\rho} = gRT \quad . \quad . \quad . \quad . \quad \text{Eq. 2.15}$$

This is the Equation of State, and a consistent system of units is :—

p = pressure (lb. per sq. ft.)
ρ = density (slugs per cu. ft.)
g = acceleration due to gravity (ft. per sec.^{2})
R = gas constant (ft. lb. per lb. per ° C. = ft. per ° C.)
T = absolute temperature (° C. + 273).

Neglecting small variations of g with location and altitude, we may write equation 2.15 in the form

$$\frac{p}{\rho} = kT \quad . \quad . \quad . \quad . \quad \text{Eq. 2.16}$$

where k is a constant $(= gR)$

Hence, if we know the density corresponding to a particular pressure and a particular temperature, we can evaluate the density for any combination of values of pressure and temperature.

2.7. Standard Atmosphere

Atmospheric conditions vary from place to place, and even from hour to hour at the same place. Since the performance of the aeroplane depends on the atmospheric conditions prevailing at the time, true comparisons of different aeroplanes are not possible unless their performance figures can be related to the same conditions of pressure, density, and temperature. For this reason a *standard atmosphere* has been agreed internationally, to which performance figures are " reduced," and on which estimates of performance are made. In this standard atmosphere the sea-level values of density, temperature, and pressure are taken to be :—

$\rho_0 = 0{\cdot}002378$ slugs per cu. ft.
$t_0 = 15°$ C. ($T_0 = 288°$ Absolute)
$p_0 = 760$ mm. of mercury
$= 29{\cdot}92$ in. of mercury
$= 2116$ lb. per sq. ft.
$= 14{\cdot}7$ lb. per sq. in.

The ratio of the density at any height h to that at sea level is known as the *relative density*, and is of such frequent occurrence in Aerodynamics that it is given a special symbol, the greek letter σ (sigma)

i.e., $$\sigma \equiv \frac{\rho}{\rho_0} \quad . \quad . \quad . \quad . \quad . \quad \text{Eq. 2.17}$$

The ratio of the pressure at height h to that at sea level is the *relative pressure*. No special symbol is used for this ratio.

$$\text{Relative pressure} = \frac{p}{p_0} \quad . \quad . \quad . \quad \text{Eq. 2.18}$$

The rate at which the temperature falls with increase of height is known as the *temperature lapse rate*. In the standard atmosphere it is taken to be $1{\cdot}981°$ C. for each 1000 ft. increase in height.
i.e.,

Temp. at height h (ft.) $\equiv t°$ C. $= 15 - 0{\cdot}001981h°$ C. Eq. 2.19

This equation is assumed to hold up to a height of 36,090 ft., beyond which height the temperature is assumed constant at $-56{\cdot}5°$ C.

The portion of the atmosphere in which the temperature follows the law given by equation 2.19 is known as the *troposphere*, the isothermal portion above this being termed the *stratosphere*.

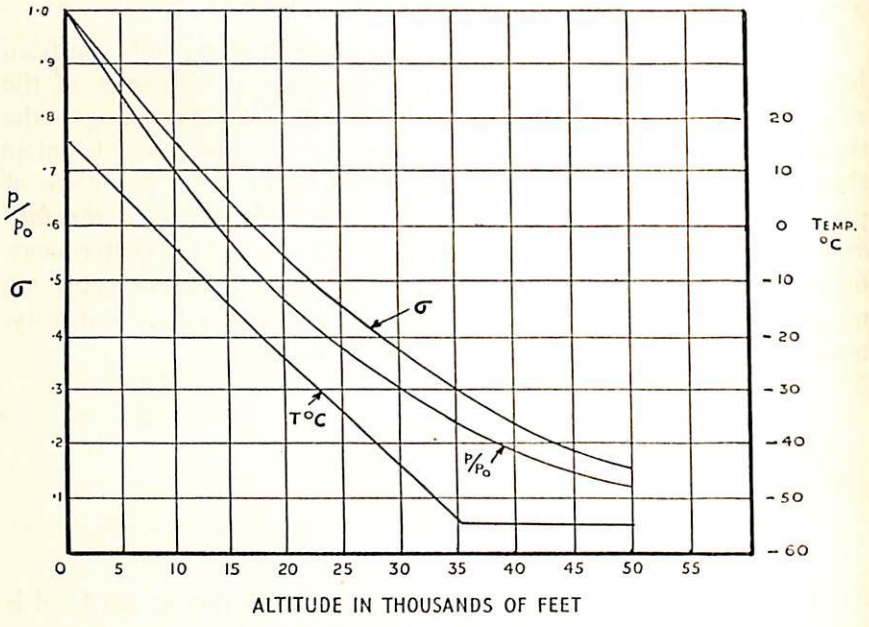


FIG. 2.4.—THE STANDARD ATMOSPHERE.

The variation of the relative density, relative pressure, and temperature, with altitude in the standard atmosphere, is shown in Table 2.1 and Fig. 2.4.

TABLE 2.1

Height (ft.).	Relative Pressure (p/p_0).	Temperature (° C.).	Relative Density ($\sigma = \rho/\rho_0$).
0	1·000	15·00	1·000
5,000	0·832	5·09	0·862
10,000	0·688	— 4·81	0·739
15,000	0·564	— 14·72	0·629
20,000	0·459	— 24·62	0·532
25,000	0·371	— 34·53	0·448
30,000	0·297	— 44·44	0·374
35,000	0·235	— 54·34	0·310

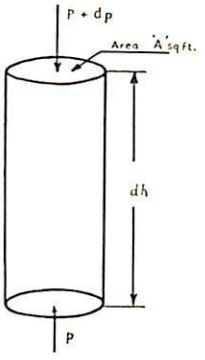


FIG. 2.5.

2.8. Relations between Properties of Standard Air

Consider the equilibrium of a small element of air at height h, of cross-sectional area A sq. ft., and thickness dh ft. (Fig. 2.5). Let the pressure at height h be p lb. per sq. ft., and at height $(h + dh)$ let it be $(p + dp)$.

The volume of the element is Adh cu. ft., and its weight is $g\rho Adh$ lb. Since the sum of the vertical forces is zero for equilibrium, we have :—

$$(p + dp)A + g\rho Adh - pA = 0$$

$$\therefore dp = -g\rho dh$$

But (equation 2.15)

$$\frac{p}{\rho} = gRT$$

$$\therefore dp = -\frac{p}{RT}dh$$

$$\frac{dp}{p} = -\frac{dh}{RT}$$

Substituting for T from equation 2.19 we have

$$\frac{dp}{p} = \frac{-dh}{R(T_0 - 0{\cdot}001981h)}$$

Integrating this from sea level ($h = 0$) to height h,

$$\log_e\left(\frac{p}{p_0}\right) = \frac{1}{R \times 0{\cdot}001981}\int_0^h \frac{-dh}{\frac{T_0}{0{\cdot}001981} - h}$$

$$= \frac{1}{96 \times 0{\cdot}001981}\log_e \frac{\frac{T_0}{0{\cdot}001981} - h}{\frac{T_0}{0{\cdot}001981}}$$

$$(T_0 = 15 + 273 = 288)$$

$$= \frac{1}{96 \times 0{\cdot}001981}\log_e\left(1 - \frac{0{\cdot}001981h}{288}\right)$$

$$= 5{\cdot}256 \log_e\left(1 - 0{\cdot}00689\frac{h}{1000}\right)$$

i.e., $$\frac{p}{p_0} = \left(1 - 0{\cdot}00689\frac{h}{1000}\right)^{5{\cdot}256} \quad \text{Eq. 2.20}$$

This may be written also

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{5{\cdot}256} \quad \text{Eq. 2.21}$$

But, from equation 2.15,

$$\frac{p}{p_0} = \frac{g\rho RT}{g\rho_0 RT_0} = \frac{\rho}{\rho_0}\cdot\frac{T}{T_0}$$

$$\therefore \frac{\rho}{\rho_0}\cdot\frac{T}{T_0} = \left(\frac{T}{T_0}\right)^{5{\cdot}256}$$

$$\therefore \quad \frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{4{\cdot}256} = \left(1 - 0{\cdot}00689\frac{h}{1000}\right)^{4{\cdot}256} \quad \text{Eq. 2.22}$$

This equation may be used to determine the relative pressure and relative density at any height up to 36,090 ft., beyond which height the temperature is presumed constant.

E.g., Determine the relative density, relative pressure and temperature at a height of 10,000 ft. in the standard atmosphere.

From equation 2.19

$$t = t_0 - 0{\cdot}001981h = 15^\circ \text{ C.} - 19{\cdot}81^\circ \text{ C.} = -\mathbf{4{\cdot}81^\circ \text{ C.}}$$

$$T_0 = 288^\circ \text{ Abs.}; \quad T = 273 - 4{\cdot}81 = 268{\cdot}19^\circ \text{ Abs.}$$

$$\frac{T}{T_0} = \frac{268{\cdot}19}{288} = \mathbf{0{\cdot}9311}$$

From equation 2.21

$$\frac{p}{p_0} = (0{\cdot}9311)^{5{\cdot}256} = \mathbf{0{\cdot}688}$$

From equation 2.22

$$\frac{\rho}{\rho_0} = \sigma = (0{\cdot}9311)^{4{\cdot}256} = \mathbf{0{\cdot}738}$$

Examples, Chapter II

1. A U-tube manometer containing liquid of specific gravity 0·8161 is used to measure the difference between two pressures. If the difference in the levels of the liquid in the two limbs of the U-tube is 1 in., calculate the pressure difference.

2. The Chattock gauge shown in Fig. 2.3 contains water, and 10 turns of the screw, which has 20 threads per in., are required to balance the difference between the two applied pressures. What is the pressure difference ?

3. In the standard atmosphere determine the density of the air at a height where the pressure is 411·5 mm. of mercury, and the temperature is $-16{\cdot}7^\circ$ C.

4. Calculate the pressure of the air in the standard atmosphere at a height where the temperature is $-22{\cdot}64^\circ$ C. and the density is 0·00131 slugs per cu. ft.

5. Using equations 2·20 and 2·22, determine the relative density and relative pressure at a height of 24,000 ft. in the standard atmosphere.

6. Repeat Ex. 5 for a height of 33,000 ft.

CHAPTER III

AIR IN MOTION

3.1. Assumptions and Definitions

(a) *Compressibility.* Air is a compressible fluid, but within limits the compression of free air by a body moving through it is negligible, and we shall make the initial assumption that we may ignore the effects of compressibility. At speeds greater than about 300 m.p.h. the effect of compressibility becomes more and more pronounced, and corrections must be applied to take this effect into account in the design of high-speed aircraft.

(b) *Viscosity.* This is a physical property possessed in varying degree by all fluids and gases ; it may be regarded, in an elementary

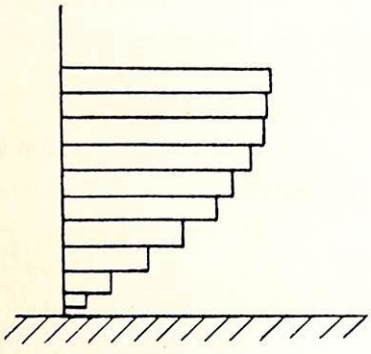


FIG. 3.1.

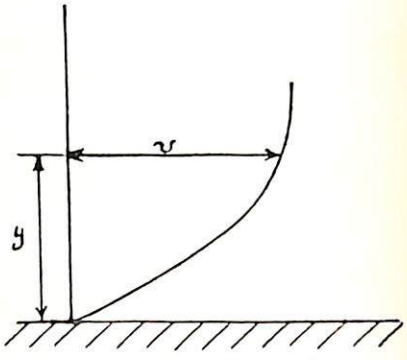


FIG. 3.2.

VELOCITY DISTRIBUTION IN BOUNDARY LAYER.

manner, as an indication of the " stickiness " of the fluid. The viscosity of air is very small, and, except in certain instances, may be neglected.

When air is flowing past the surface of a solid body, the particles of air immediately adjacent to the surface stick to it, and are brought to rest relative to the body. That this is so is verified by the fact that dust is not removed from the body of a car or the wings of an aeroplane by the airflow resulting from motion of the car or aeroplane. Viscosity causes layers of air further from the surface to be slowed up, so that the velocity of different layers

will vary somewhat as indicated in Fig. 3.1, where the length of lines represents the velocity of the corresponding layer of air. By making the layers of air very thin, this diagram would ultimately become the curve of velocity distribution near the surface, as in Fig. 3.2, in which y represents distance from the surface and v the velocity. At some distance from the surface the effects of viscosity will disappear, and the velocity will be that of the undisturbed air stream. The portion of the fluid in which the velocity is changing as the distance from the surface increases is known as the *boundary layer*, and it will be necessary later to consider the

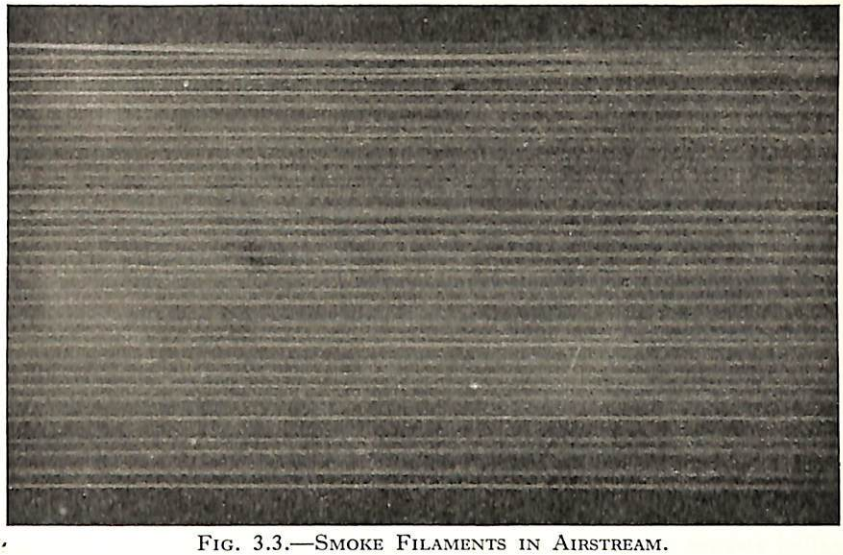


FIG. 3.3.—SMOKE FILAMENTS IN AIRSTREAM.

flow in this region in greater detail. For the present it suffices to realise that, in order to produce the changes of velocity in the boundary layer, there must be a force acting on each layer of air parallel and opposite to the direction of motion. These "sliding" or "shearing" forces are known as "viscous forces," and their magnitudes depend on the rate at which one layer is sliding over the next. The total thickness of the boundary layer is very small—near the leading edge of a wing it may be only a few thousandths of an inch thick, and it will gradually increase in thickness towards the trailing edge—so that the velocity increases from zero at the surface to the full value of the undisturbed stream in a very short height. This rapid increase in velocity with distance from the

surface gives rise to large viscous forces near the surface, which provide a considerable part of the resistance which a body offers to motion through the air—a part known as *skin friction.*

For the time being, however, we shall consider the flow of air outside the boundary layer and we shall neglect the effects of viscosity —i.e., in what follows, unless any statement is made to the contrary, air is assumed to be incompressible and non-viscous.

(c) *Stream Tubes, Streamlines and Path Lines.* If we introduce thin filaments of smoke into a steady stream of air we obtain a picture like Fig. 3.3. We may regard these filaments of smoke as thin tubes along which particles of air are flowing as indicated in Fig. 3.4. Each of these tubes is such that the air in it flows parallel to the axis of the tube and no air flows across the tube walls. Whilst the introduction of smoke makes these tubes visible, it is possible to

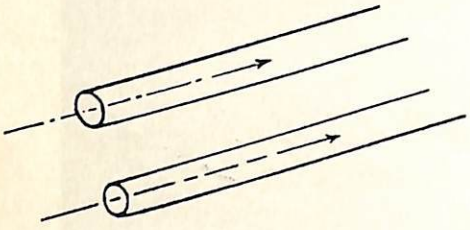


FIG. 3.4.—STREAM TUBES.

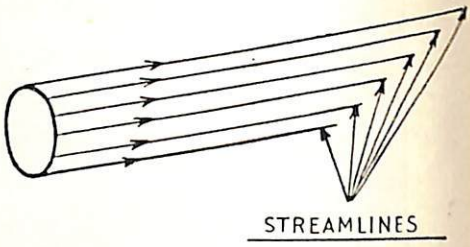


FIG. 3.5.—STREAM TUBE BOUNDED BY STREAMLINES.

imagine in a stream of air tubes which are so disposed that there is no flow across them, only flow along their axes. Such tubes are called *stream tubes.* If, now, we imagine any such tube to become very, very thin, so that ultimately we can regard it as a line, we have a *streamline.* It follows that a streamline is a line drawn so that the flow at any point on it is along the line, and there is no flow across it anywhere.

Alternatively, we could have started with the definition of a streamline and said that a stream tube is a tube whose boundaries are composed of streamlines (Fig. 3.5).

The picture obtained by drawing the streamlines is known as the *flow pattern,* and it should be realised at once that the flow pattern may change from instant to instant. A set of streamlines showing the direction of the velocity at all points in the field of flow will exist at any given time, but an entirely different set may appear

afterwards. Consequently, the streamlines do not, in general, show the paths followed by individual particles of fluid—lines which do this are *path lines.*

With our assumption of incompressibility, and the fact that there is no flow across a streamline, we may use the streamline pattern to indicate the variation of the magnitude of velocity in the field of flow. For example, consider the two adjacent streamlines in Fig. 3.6. It will be noticed that the streamlines are drawn closer together at B than at A. The fluid is incompressible so that the same amount of fluid must pass each point in unit time, no fluid being able to flow across the boundary lines. Hence the velocity must be greater at B than at A. Consequently, to indicate an increase in velocity, we draw the streamlines more closely together, and vice versa.

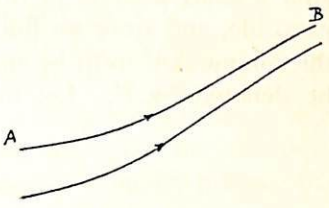


FIG. 3.6.—SPACING OF STREAMLINES.

(d) *Steady Motion and Unsteady Motion.* If the flow pattern does not change from one instant to another but remains constant, the motion is said to be "steady." In this type of motion the streamlines and path lines are the same.

When the flow pattern is changing from one instant to another we say the motion is "unsteady" or eddying, and in this case the streamlines and path lines are entirely different.

Other types of motion, particularly with reference to the boundary layer, will be discussed later.

3.2. Bernoulli's Theorem

In Chapter II, mention was made of the pressure of air at rest. Air in motion also exerts pressure on any surface with which it is in contact, but this pressure is not now the same in all directions at any one point. It can, however, be split into two parts—(i) the *static pressure*, denoted by p, which is the same in all directions at a given point, and acts at right-angles to a surface in contact with the airstream, and (ii) the *dynamic pressure* (equal, as we shall see later, to $\frac{1}{2}\rho v^2$, where ρ is the density of the air, and v the stream velocity), which acts in the direction of motion.

In 1738, Daniel Bernoulli enunciated a theorem of great importance in Fluid Mechanics, connecting the static pressure and dynamic pressure of a fluid in motion. For the present, making the assump-

tions outlined in paras. 3.1 (a) and (b), and leaving the more detailed exposition until a later stage, we consider only the simplest form of the theorem, viz., along any streamline in steady flow of an incompressible, inviscid fluid the sum of the pressure energy, kinetic energy and potential energy is constant. In general, changes in potential energy will be negligible in aerodynamics, so that we need to consider only the sum of the pressure energy and the kinetic energy.

Now, Fig. 3.7, consider a stream tube in a steady flow and assume that the fluid which at one instant is at AB moves to A'B' after a short interval of time. Since the fluid is assumed incompressible, and since no fluid can flow across the walls of the tube, the volume AA' must be equal to the volume BB'. Let this volume be denoted by V. Let the pressure, velocity and cross-sectional

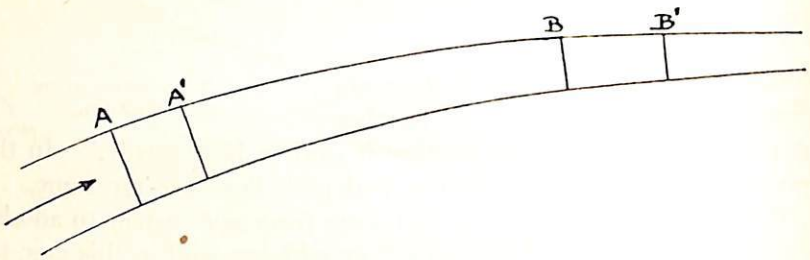


FIG. 3.7.—BERNOULLI'S THEOREM.

area of the tube at A be p_A, v_A, S_A, respectively, and let p_B, v_B, S_B represent similar quantities at B. The pressure at A does work on AB by moving fluid from A to A'. At the same time work is done against the pressure at B by movement of fluid from B to B'. The force acting on the fluid at A in the direction of motion is $p_A S_A$, provided we can assume the stream tube to be sufficiently narrow for the pressure to be regarded as constant over the cross-section. The work done by this force in moving through the distance AA' is

$$p_A S_A \times AA' = p_A \times V$$

Similarly, the work done *against* the force at B is

$$p_B S_B \times BB' = p_B \times V$$

Hence, the nett work done on AB $= p_A V - p_B V = (p_A - p_B)V$. This work is used in changing the kinetic energy.

Since mass is density $\times$ volume, the change in kinetic energy,

equal to the difference between the kinetic energy of BB′ and that of AA′, is

$$\tfrac{1}{2}\rho V(v_B^2 - v_A^2)$$

Hence, as work done equals change in kinetic energy,

$$(p_A - p_B)V = \tfrac{1}{2}\rho V(v_B^2 - v_A^2)$$

i.e.,
$$p_A + \tfrac{1}{2}\rho v_A^2 = p_B + \tfrac{1}{2}\rho v_B^2$$

Since A and B are any two points along the stream tube, it follows that along the tube,

$$p + \tfrac{1}{2}\rho v^2 = \text{constant} \quad . \quad . \quad . \quad \text{Eq. 3.1}$$

where p = static pressure of the air (lb. per sq. ft.)
ρ = density of the air (slugs per cu. ft.)
v = velocity of the airstream (ft. per sec.).

The units given for p, ρ, v, are those commonly used in this country. Any other system could be used, of course, but the units chosen must be consistent—for example, if p is in lb. per sq. ft., v must be in ft. per sec., *not* m.p.h.

It will be observed that in equation 3.1, the cross-sectional area of the tube does not appear, so that the equation holds when the tube becomes indefinitely thin—i.e., along a streamline. *Bernoulli's equation*, as equation 3.1 is known, is merely a statement of the *principle of conservation of energy*, and in the simple form derived above variations of other types of energy than pressure energy and kinetic energy are ignored. This simplification is possible only if we make the assumptions outlined earlier.

An immediate deduction from equation 3.1 is that an increase in static pressure along a streamline must be accompanied by a fall in speed and vice versa.

3·3. Stagnation Point and Stagnation Pressure

When air flows past a solid body, as in Fig. 3.8, there must be a dividing streamline such as AB, which ends on the body, the velocity at B being zero.

Applying Bernoulli's equation to the streamline AB, we have

$$p_A + \tfrac{1}{2}\rho v_A^2 = p_B + \tfrac{1}{2}\rho v_B^2$$

i.e.,
$$p_B = p_A + \tfrac{1}{2}\rho v_A^2 \text{ since } v_B = 0 \quad . \quad . \quad \text{Eq. 3.2}$$

The point at B at which the stream is brought to rest is known as a *stagnation point*, and the amount by which the static pressure

there (p_B) exceeds the static pressure of the undisturbed stream (p_A) is called the *stagnation pressure.*

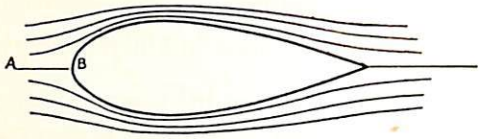


FIG. 3.8.—STAGNATION POINT.

The difference in pressure between any two points A and B on the same streamline is

$$p_B - p_A = \tfrac{1}{2}\rho v_A^2 - \tfrac{1}{2}\rho v_B^2$$

This pressure difference, obviously, has its greatest value when v_B is zero, so that the stagnation pressure ($\frac{1}{2}\rho v^2$) is the greatest pressure increase that can be obtained in a stream of velocity v.

The quantity $\frac{1}{2}\rho v^2$ is often referred to also as the *dynamic pressure*, and it is of such frequent occurrence in aerodynamics that it is given a special symbol, q.

3·4. Application of Bernoulli's Theorem to Airspeed Measurement

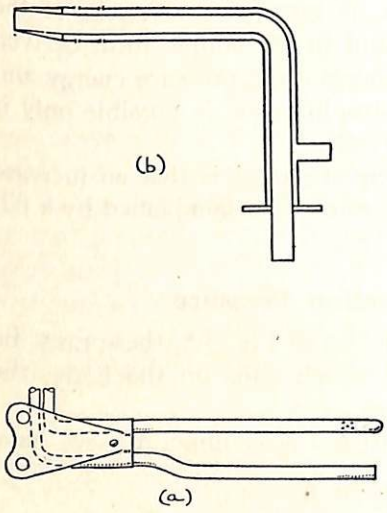


FIG. 3.9.—PITOT-STATIC TUBES.

(i) *Pitot-Static Tube.* Alternative forms of this instrument are shown in Fig. 3.9. It consists essentially of two tubes facing into the airstream, and connected to opposite sides of some form of pressure gauge. The tubes may be separated from each other as in Fig. 3.9 (a), or concentric as in Fig. 3.9 (b). One tube, the *pitot tube*, is open ended, the other, the *static tube*, has a closed end and a series of small holes drilled round its circumference as indicated. These holes may be replaced by an annular slot in some types.

Provided certain conditions are fulfilled concerning the size and position of the holes, the pressure in the static tube will be the static pressure of the undisturbed stream p_0.

If the two tubes are connected to opposite sides of a pressure gauge, there will be no flow through the pitot tube, so that there will be a stagnation point near the mouth of this tube. By equation 3.2, the pressure in the pitot tube (known as the *pitot head* or *total head*), will be $p_0 + \frac{1}{2}\rho v^2$. The pressure difference indicated by the gauge will be, therefore,

$$(p_0 + \tfrac{1}{2}\rho v^2) - p_0 = \tfrac{1}{2}\rho v^2$$

From a knowledge of the difference between the pitot pressure and the static pressure, it is possible, given the density of the air, to calculate the speed of the airstream.

E.g., A pitot-static tube is placed in an airstream and connected to a U-tube manometer. The pressure difference is obtained as 3 cm. of methylated spirits (specific gravity 0·816). Assuming the density of the air is 0·00238 slugs per cu. ft., calculate the velocity of the airstream.

3 cm. meths. is equivalent to $\frac{3}{2{\cdot}54}$ in. meths.

$$= \frac{3}{2{\cdot}54 \times 12} \text{ ft. meths.}$$

$$= \frac{3}{2{\cdot}54 \times 12} \times (0{\cdot}816 \times 62{\cdot}5) \text{ lb. per sq. ft.}$$

(since 1 cu. ft. water weighs 62·5 lb.)

$$= \mathbf{5{\cdot}02} \text{ lb. per sq. ft.}$$

$$\therefore \tfrac{1}{2}\rho v^2 = 5{\cdot}02 \text{ lb. per sq. ft.}$$

$$v^2 = 5{\cdot}02 \times \frac{2}{0{\cdot}00238} \text{ (ft. per sec.)}^2$$

$$\therefore \quad v = \mathbf{65} \text{ ft. per sec.}$$

The combination of the pitot-static tube and airspeed indicator is the most commonly used method of measuring airspeed on aeroplanes. The airspeed indicator is a form of pressure gauge in which pressure changes cause movement of a diaphragm, this movement in turn being transferred by gearing to the movement of a needle round a dial, which is graduated to read directly in m.p.h. or knots.

(ii) *Venturi Tube.* With the pitot-static tube, the airspeed is calculated from a measurement of the difference between the static pressure and a pressure greater than static. With the *venturi tube*

the pressure difference utilised is that between static pressure and a pressure less than static. A venturi tube is shown diagrammatically in Fig. 3.10. From the entrance, where the diameter is D, the tube converges to a diameter d at the throat, beyond which it expands again.

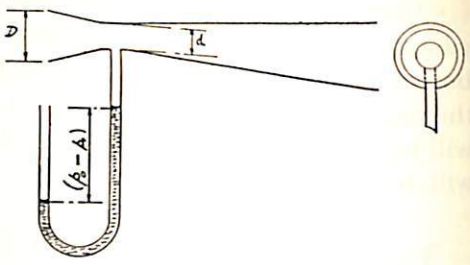


FIG. 3.10.—VENTURI TUBE.

The pressure outside the tube is the static pressure of the undisturbed stream, p_0. At the throat let the static pressure be p_1, and the velocity v_1. Then, if v_0 be the velocity of the undisturbed stream, along a streamline passing through the tube,

$$p_0 + \tfrac{1}{2}\rho v_0^2 = p_1 + \tfrac{1}{2}\rho v_1^2$$

Assuming incompressible flow, we must have

$$\frac{\pi D^2}{4} v_0 = \frac{\pi d^2}{4} v_1$$

since the same quantity of air flows in unit time past the two sections.

$$\therefore \frac{v_1}{v_0} = \frac{D^2}{d^2} = K \text{ (say)}$$

Substituting for v_1 in Bernoulli's equation,

$$p_0 + \tfrac{1}{2}\rho v_0^2 = p_1 + \tfrac{1}{2}\rho K^2 v_0^2$$

i.e.,

$$p_0 - p_1 = \tfrac{1}{2}\rho v_0^2 (K^2 - 1) \quad . \quad . \quad \text{Eq. 3.3}$$

so that measurement of the pressure difference $p_0 - p_1$ enables us to compute v_0, when the density of the airstream is known.

From equation 3.3,

$$v_0 = \left[\frac{p_0 - p_1}{\tfrac{1}{2}\rho (K^2 - 1)} \right]^{\frac{1}{2}}$$

A venturi tube is suitable for measurement of the speed of fluid flow only in circumstances in which it is constrained to run full, for example, when the venturi forms part of a pipe line through which fluid is flowing. When the tube is exposed in a free airstream, however, some of the oncoming air is diverted ahead of the tube, and there is no constraint to ensure that the tube runs full. Consequently the venturi tube is not regarded as a reliable means of airspeed measurement for aeroplanes.

3.5. True Airspeed and Indicated Airspeed

We have seen that when a pitot tube and a static tube are connected to opposite sides of a pressure gauge, the pressure difference is equal to $\frac{1}{2}\rho v^2$. If the gauge is calibrated so that a pressure difference is read directly as m.p.h., as on the airspeed indicator, it will be obvious that the calibration can be correct for only one particular value of density ρ. It is usual to calibrate an airspeed indicator for the density corresponding to standard atmosphere conditions at sea level ($\rho_0 = 0{\cdot}00238$ slugs per cu. ft.), so that the reading on the indicator dial gives the true value of the airspeed only when the density has this value. For all other conditions the reading requires correction to give the *true airspeed*. The reading on the indicator is referred to as the *indicated airspeed*, and is denoted by v_i. Then, if v_T be the true airspeed and ρ the true density of the air, the pressure difference transmitted to the indicator is equal to $\frac{1}{2}\rho v_T^2$. The pressure difference corresponding to a reading v_i on the indicator dial is $\frac{1}{2}\rho_0 v_i^2$, since the dial is calibrated for air at sea level density, so that the relationship between the true airspeed and the indicated airspeed is

$$\frac{1}{2}\rho v_T^2 = \frac{1}{2}\rho_0 v_i^2$$

$$\therefore v_T = v_i\sqrt{\frac{\rho_0}{\rho}} = v_i \times \frac{1}{\sqrt{\sigma}} \quad . \quad . \qquad \text{Eq. 3.4}$$

E.g., The speed of an aeroplane at 10,000 ft. is 160 m.p.h. A.S.I. What is the true airspeed?

At 10,000 ft. the value of the relative density is $\sigma = 0{\cdot}739$

$$\therefore v_T = \frac{v_i}{\sqrt{\sigma}} = \frac{160}{\sqrt{0{\cdot}739}} = \mathbf{186} \text{ m.p.h.}$$

Note on Airspeed. It is essential to understand that when we speak of the speed of an aeroplane, we refer usually to airspeed, i.e., the speed of the aeroplane relative to the air—not to ground speed. For example, an aeroplane flying at a true airspeed of 300 m.p.h. into a headwind of 40 m.p.h. would have a ground speed of 260 m.p.h. It is quite conceivable for an aeroplane to have airspeed but zero groundspeed.

3.6. Errors arising from use of Pitot-Static Tube

When a pitot-static tube is used in combination with an airspeed indicator to measure the airspeed of an aeroplane, there are several

sources of error for which allowance must be made. The most important of these are as follows:

1. *Position Error.* For best results the pitot-static tube should be parallel to the undisturbed stream. Since the attitude of an aeroplane does not remain constant relative to the undisturbed airstream, it is not possible to fulfil this requirement. Furthermore, the airflow past the pitot-static tube may be influenced by the proximity of parts of the aeroplane, such as the wings.

The error arising from these factors is known as *position error*, and correction must be obtained for all aeroplanes.

2. *Instrument Error.* This is an error arising in the graduation of the airspeed indicator dial, and each instrument is calibrated against a standard to provide the necessary corrections.

3. *Compressibility Error.* At high speeds the difference between the pitot and static pressures is not equal to $\frac{1}{2}\rho v^2$, and the reading should be corrected for this effect. Table 3.1 gives an indication of the magnitude of this error at various speeds.

TABLE 3.1

Speed, m.p.h. . .	100	150	200	250	300	350	400	450
Error, % . . .	0·444	1·0	1·77	2·78	4·0	5·44	7·11	9·0

Throughout the text, indicated airspeed will be assumed to be fully corrected for the above errors.

Examples, Chapter III

1. What is a streamline? How does the flow pattern indicate variations in velocity?

2. A pitot-static tube is placed in an airstream and connected to a Chattock gauge of dimensions as indicated in Fig. 2.3. If the pitch of the screw is 20 threads per in., and if 6 turns are required for balance, the liquid in the gauge being water, determine the speed of the airstream.

3. The air passing over an aeroplane wing has its speed increased from 135 m.p.h. at one point to 250 m.p.h. at another point. What is the difference in static pressure between the two points, assuming the air density to be 0·00238 slugs per cu. ft.?

4. A model wing is mounted in an airstream of 40 ft. per sec. A manometer is set up so that it measures the difference between the static pressure of the undisturbed stream and the static pressure at a point on the upper surface of the wing. If the pressure at the point on the wing is lower than that of the undisturbed stream by 1 cm. of meths. (specific gravity 0·816), calculate the increase in speed at this point.

5. Calculate the stagnation pressure on an aeroplane wing at a speed of 210 m.p.h. at sea level, and at 10,000 ft.

6. An aeroplane climbs at a constant indicated airspeed of 120 m.p.h. from sea level to a height of 15,000 ft. What is the true airspeed at heights of 5000 ft., 10,000 ft., 12,000 ft. and 15,000 ft. ?

CHAPTER IV

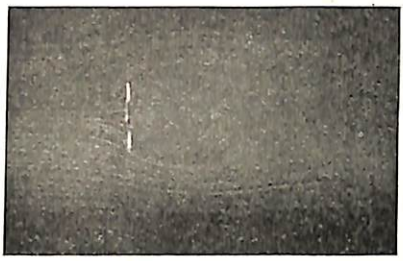
AEROFOILS

4.1. Flow past Solid Bodies

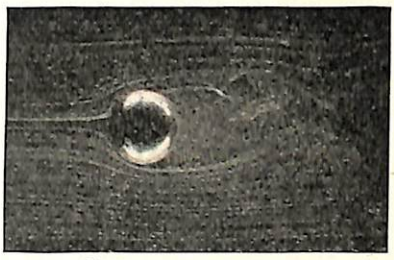
ALTHOUGH in aircraft design we are concerned with solid bodies moving through air (which may be assumed to be still air), for simplicity in developing the theory of aerodynamics it is convenient to consider what happens when air flows past a stationary body. The aerodynamic forces will be the same in both cases if the relative speed is the same.

We have stated in para. 2.2, that the pressure at a point in a stationary fluid is the same in all directions. For a small body immersed in a large mass of stationary fluid, this means that the pressure at all points of the body is approximately of constant value, and hence there will be no resultant force due to pressure. However, if the body is moving through the fluid, or the fluid flowing past the body, the pressure distribution changes and a resultant force is produced, which may be parallel to the direction of motion or perpendicular to the direction of motion, or intermediate between these two. In general the resultant force may be resolved into two components, one perpendicular to the direction of the undisturbed stream, known as the *lift*, the other parallel to the undisturbed flow, known as the *drag*.

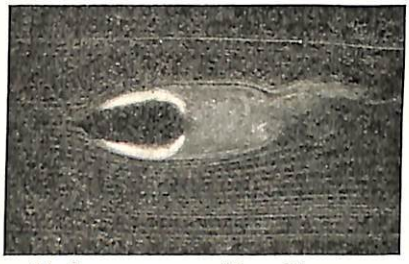
Let us consider briefly the *drag* force in relation to the pattern of the flow past a body. In Fig. 4.1. are shown smoke tunnel photographs of the flow past several bodies, all of which have the same frontal area. The measured drag, in pounds, at a speed of 55 ft. per sec. is quoted for each one. It will be readily observed that a decrease in drag is associated with a narrower " wake " of eddying flow behind the body, the least drag being obtained with a body shaped so that the streamlines follow the contour quite closely. *Streamlining*, or *fairing*, is the name given to the shaping of a body so that the formation of a wide wake of eddying flow is avoided. We shall see later that the formation of this wake is not the only factor affecting the drag force. Contributions arise also from the tangential forces due to the viscosity of the air (skin-



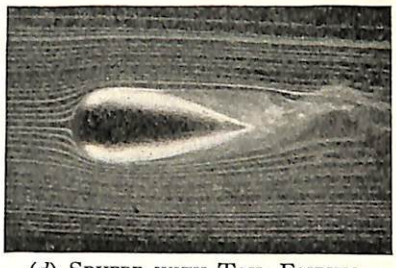
(*a*) FLAT PLATE. 0·205 lb.



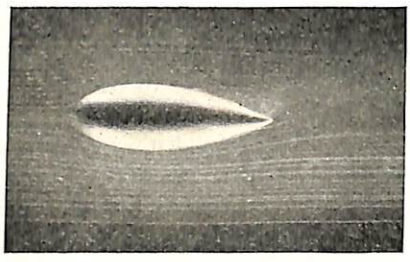
(*b*) SPHERE. 0·093 lb.



(*c*) SPHERE WITH NOSE FAIRING. 0·074 lb.



(*d*) SPHERE WITH TAIL FAIRING. 0·025 lb.



(*e*) STREAMLINED BODY. 0·022 lb.

FIG. 4.1.—FLOW PAST BODIES OF THE SAME FRONTAL AREA.
The figures give the drag of each model at a speed of 55 ft. per sec.

friction drag) and from the consideration of three-dimensional flow. Detailed discussion of the items making up the total drag will, however, be postponed until a later stage.

4.2. The Flat Plate—Resistance Coefficient

Consider a flat plate normal to the airstream, as in Fig. 4.2. The plate will experience a force R in the direction of flow, as indicated. Theory indicates and experiment confirms that R is proportional to the area of the plate (S), the density of the air (ρ), and the square of the speed (V).

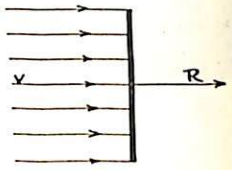


FIG. 4.2.—FLAT PLATE NORMAL TO AIRSTREAM.

Hence, we may write $R = k_R \rho S V^2$ where k_R is a constant.

If we use $C_R = 2k_R$, this relation becomes

$$R = C_R \cdot \tfrac{1}{2}\rho S V^2 = C_R S q \quad . \quad . \quad \text{Eq. 4.1}$$

When precise results are required it is necessary to take account of the fact that C_R, the resistance coefficient, is not truly constant, but for present purposes we may neglect the variation of C_R and assume it to have the constant value of approximately 1·28.

Hence, the drag of a flat plate, 3 in. diameter, normal to an airstream of 100 ft. per sec. will be

$$R = 1{\cdot}28 \times \tfrac{1}{2} \times 0{\cdot}00238 \times \frac{\pi}{4} \times \frac{9}{144} \times 100^2$$

$$= 7{\cdot}49 \text{ lb.}$$

4.3. Lift and Drag Coefficients

If the flat plate is now inclined at angle α to the oncoming stream, as in Fig. 4.3, the resultant force is no longer at right angles to the plate, being inclined at a small angle γ from the normal. We may still write $R = C_R S q$, although the value of C_R will now vary with α.

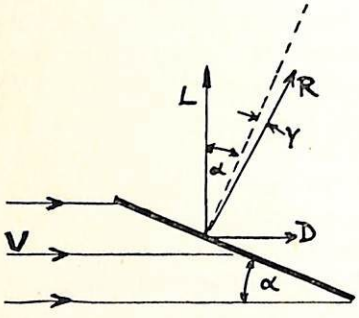


FIG. 4.3.—FLAT PLATE INCLINED TO AIRSTREAM.

The existence of this upwards force may be deduced also from consideration of the flow pattern past the plate (Fig. 4.4). It will be observed that on passing the plate the streamlines are deflected down-

wards, indicating that the plate is exerting a downwards force on the airstream. Since to every action there is an equal and opposite reaction, it follows that the airstream must exert an upwards force on the plate.

Resolving R into two components perpendicular and parallel to the airstream,

$$\text{Lift} \equiv L = R \sin(\gamma + \alpha) = C_R \cos(\gamma + \alpha) Sq$$
$$\text{Drag} \equiv D = R \sin(\gamma + \alpha) = C_R \sin(\gamma + \alpha) Sq$$

$C_R \cos(\gamma + \alpha)$ and $C_R \sin(\gamma + \alpha)$ are non-dimensional coefficients which depend on α (γ depends on α), and we may write

$$C_R \cos(\gamma + \alpha) \equiv C_L$$
and $$C_R \sin(\gamma + \alpha) \equiv C_D$$
so that $$L = C_L Sq \quad \text{and} \quad D = C_D Sq \quad \text{Eqs. 4.2}$$

C_L and C_D are known as the *lift coefficient* and *drag coefficient* respectively.

4.4. Evolution of the Aerofoil

Although a flat plate may be used as a lifting surface it is not very efficient, and it has the disadvantage of not providing sufficient thickness for structural purposes. Furthermore, examination of the flow past a flat plate inclined at a larger angle to the airstream reveals a very early breakaway of the flow over the upper surface, which results in a loss of lift and a large increase in drag. By curving the leading edge as in Fig. 4.5, a smooth flow can be maintained up to a larger angle of inclination, with an increase in the efficiency of the plate as a lifting surface. Reference to Fig. 4.6, however, where we see the curved plate at a small inclination to the stream, shows a region of eddying flow behind the leading edge on the lower surface, the airstream being unable to conform to the curvature of the underside of the plate. This, again, results in an increase in drag. To eliminate this defect, the underside of the curved plate is filled in as in Figs. 4.7 and 4.8, resulting in what we now term an aerofoil section. It will be noted that the flow has been improved, and, at the same time, depth has been provided to accommodate structural members.

Many different shapes of aerofoil sections have been proposed and investigated. No single section exists which is suitable for all types of aeroplanes in every respect, and the designer must always compromise between the various requirements in selecting an aero-

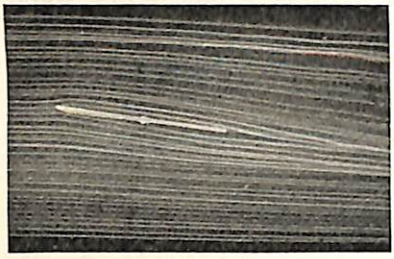


FIG. 4.4.—FLAT PLATE AT MODERATE ANGLE OF ATTACK.

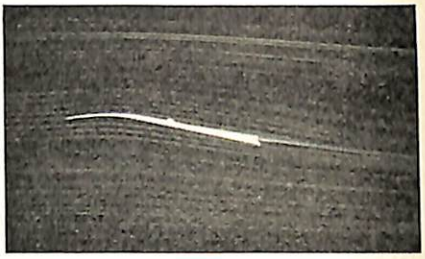


FIG. 4.5.—PLATE WITH CURVED LEADING EDGE AT MODERATE ANGLE OF ATTACK.

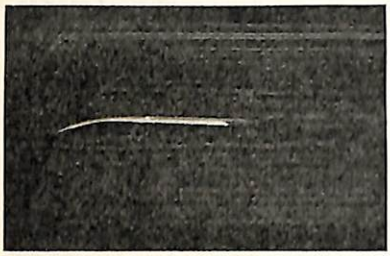


FIG. 4.6.—PLATE WITH CURVED LEADING EDGE AT LOW ANGLE OF ATTACK.

FIG. 4.7.—AEROFOIL SECTION AT LOW ANGLE OF ATTACK.

FIG. 4.8.—AEROFOIL SECTION AT MODERATE ANGLE OF ATTACK.

FIGS. 4.4–4.8.—EVOLUTION OF THE AEROFOIL.

foil for a particular aeroplane. To assist him in making his choice, intensive tests have been carried out on a large number of sections, and a representative set of data resulting from such tests on one aerofoil is given later in this chapter, together with some comments

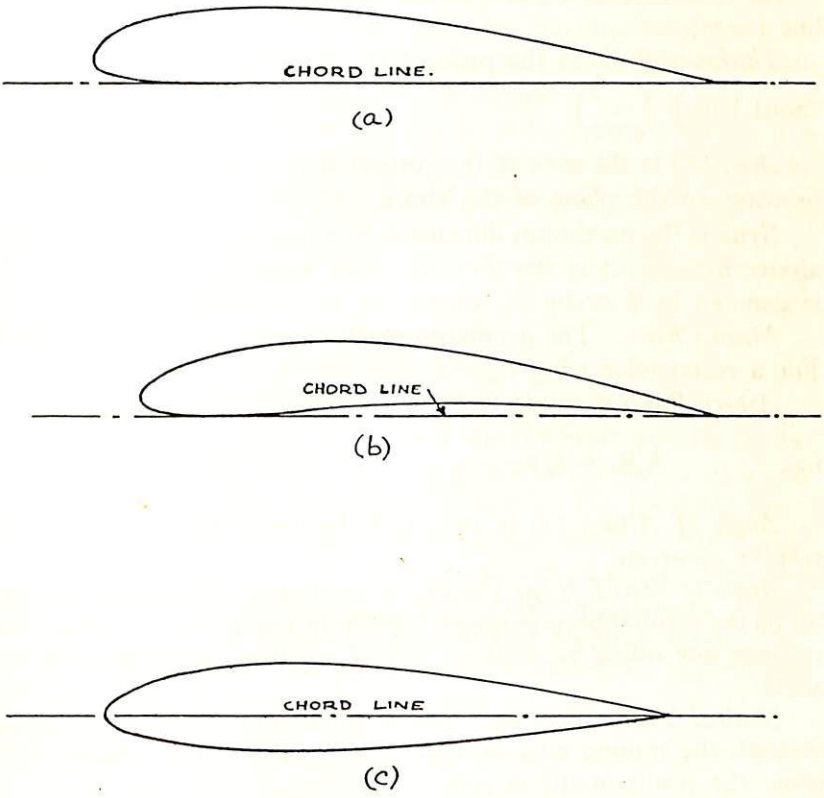


FIG. 4.9.—CHORD LINE CONVENTIONS.

4.5. Definitions relating to Aerofoils

Chord is a reference line for the aerofoil section. Conventions usually adopted for various shapes are shown in Fig. 4.9. In the case of the bi-convex aerofoil (Fig. 4.9 (c)), the chord is usually taken as the line joining the centres of curvature of the leading and trailing edges (these two latter terms are self-explanatory).

Chord Length (c) is the projection of the aerofoil section on the chord line.

Camber at a point is the perpendicular distance of that point from the chord. It is usually expressed in percentage of the chord length.

Mean Camber is the camber of any point on the line equidistant from the upper and lower surfaces (mean or medial line).

Maximum Mean Camber is the maximum camber of the mean line ($\equiv m$).

Thickness Ratio is the ratio of the maximum thickness to the chord length $\left(=\frac{t}{c}\right)$.

Area (S) is the area of the projected plan form of the aerofoil or wing on the plane of the chord.

Span is the maximum dimension in a plane perpendicular to the airstream; i.e., it is the distance from wing tip to wing tip. It is denoted by b or by $2s$, where s is the semi-span.

Mean Chord. The geometric mean chord (c_m) is the ratio S/b. For a rectangular wing $c_m = c$.

Aspect Ratio is equal to span divided by mean chord,

i.e., $$\text{A.R.} = b/c_m = \frac{b}{S/b} = \frac{b^2}{S} = (\text{Span})^2/\text{Area}$$

Angle of Attack (α) is the angle between the chord and the relative airstream.

Angle of Zero Lift (α_{L_0}) is the value of α at which the resultant lift on the aerofoil becomes zero. With the exception of symmetrical sections, for which $\alpha_{L_0} = 0$, in general α_{L_0} will be a small negative angle.

Neutral Line or Zero Lift Line is usually defined as a line drawn through the trailing edge so that it is parallel to the relative wind when the resultant lift is zero.

Absolute Angle of Attack (α_a) is the true aerodynamic angle of attack—the angle between the neutral line and the relative wind.

$$\alpha_a = \alpha - \alpha_{L_0}$$

e.g., for the section R.A.F. 38, $\alpha_{L_0} = -2{\cdot}0°$ so that the absolute angle of attack when $\alpha = 10°$ will be

$$\alpha_a = \alpha - \alpha_{L_0} = 10° - (-2{\cdot}0°) = 12{\cdot}0°$$

It should be noted that when experimental data are given for an aerofoil section, the angle of attack quoted is that denoted above by α.

Before we proceed to study the results of experiments on a particular aerofoil, it is necessary to define two more coefficients which we shall use.

4.6. Centre of Pressure Coefficient

A force is completely defined if we specify its magnitude, direction and line of action. If we know the magnitudes of the two components, L and D, of the resultant force R we know the first two of the required quantities. To determine the line of action we need to know some point on that line, and for convenience we use the point at which the line of action of R cuts the chord of the aerofoil. This point is called the *centre of pressure*, usually abbreviated to C.P.

If the C.P. is distant x from the leading edge, measured along the chord whose length is c, then we define a *centre of pressure coefficient*, $C_{C.P.}$, as equal to the ratio x/c.

4.7. Moment Coefficient

It is not always necessary to determine the exact position of the C.P., and test results more often quote a *moment coefficient* about some chosen point. If M be the moment of the resultant force about this point, we may write $M = C_M Sqc$, where C_M is a non-dimensional coefficient. The length c is introduced because the product $C_M Sq$ represents force, and to obtain moment whilst retaining C_M as a non-dimensional coefficient requires this product to be multiplied by a length. The relation between the various coefficients may be demonstrated simply as follows.

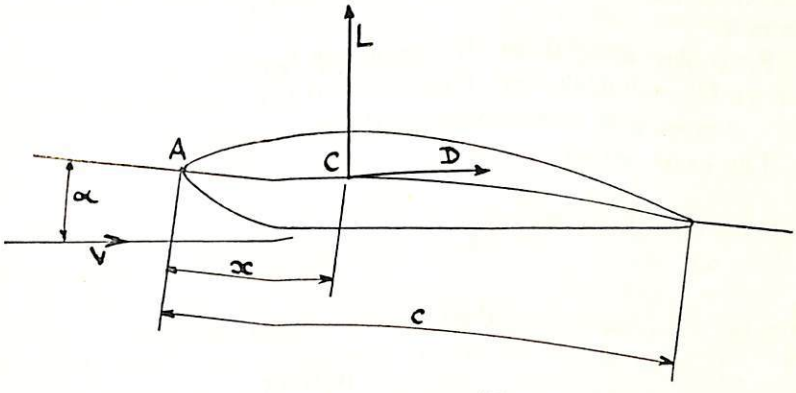


FIG. 4.10.—CENTRE OF PRESSURE AND MOMENT COEFFICIENT.

In Fig. 4.10 let L and D be the lift and drag, acting at the centre of pressure, C, of the aerofoil shown. Let C be at distance x from the leading edge, A, and let $M_{L.E.}$ be the moment about A of the resultant force and, hence, also the sum of the moments about A of L and D. We take as positive a moment which raises the leading edge.

Then $$M_{L.E.} = -(L \times x\cos\alpha + D \times x\sin\alpha)$$
$$= -(C_L Sq \times x\cos\alpha + C_D Sq \times x\sin\alpha)$$
$$= -xSq(C_L\cos\alpha + C_D\sin\alpha)$$

But $$x = c \times C_{C.P.}$$

Hence, dividing both sides of the equation by Sqc,

$$\frac{M_{L.E.}}{Sqc} = -C_{C.P.}(C_L\cos\alpha + C_D\sin\alpha) \quad . \qquad \text{Eq. 4.3}$$

The expression on the right is non-dimensional, and may be written $C_{M_{L.E.}}$

Then $$\frac{M_{L.E.}}{Sqc} = C_{M_{L.E.}}$$

or $$M_{L.E.} = C_{M_{L.E.}}Sqc \quad . \quad . \quad . \qquad \text{Eq. 4.4}$$

$C_{M_{L.E.}}$ is the moment coefficient about the leading edge.

In general, $\cos\alpha$ will be nearly 1, and $C_D\sin\alpha$ will be very small compared with C_L, so that

$$C_{M_{L.E.}} = -C_{C.P.}(C_L\cos\alpha + C_D\sin\alpha) \simeq -C_{C.P.} \times C_L$$

or $$C_{C.P.} \simeq -\frac{C_{M_{L.E.}}}{C_L} \quad . \quad . \quad . \qquad \text{Eq. 4.5}$$

The closeness of this approximation is, perhaps, best shown by an example.

E.g., An aerofoil at 5° angle of attack has $C_L = 0{\cdot}488$, $C_D = 0{\cdot}0248$ and $C_{M_{L.E.}} = -0{\cdot}131$. Find the C.P. by exact and approximate methods.

The exact solution is :—

$$C_{C.P.} = -\frac{C_{M_{L.E.}}}{C_L\cos\alpha + C_D\sin\alpha}$$
$$= -\frac{-0{\cdot}131}{0{\cdot}488\cos 5 + 0{\cdot}0248\sin 5}$$
$$= +\frac{0{\cdot}131}{0{\cdot}48816} = \mathbf{0{\cdot}2683}$$

The approximate solution is:—

$$C_{C.P.} = -\frac{C_{M_{L.E.}}}{C_L}$$

$$= -\frac{-0{\cdot}131}{0{\cdot}488} = \mathbf{0{\cdot}26843}$$

In this particular example the approximation is very close.

4.8. Characteristic Curves of an Aerofoil

For purposes of comparison of different aerofoil sections, and in order to enable a designer to choose a section suitable to his requirements, the variations of the coefficients defined above are obtained from tests and presented in the form of characteristic curves for each section. Typical curves are shown in Figs. 4.11.

Certain properties of these curves are common to most sections, and will be discussed below.

$C_L \sim \alpha$ Curve

We note, first of all, that C_L becomes zero not when α is 0° but when $\alpha = -2°$. The fact that C_L is zero does not mean that there are no lift forces acting on the aerofoil; it indicates that the sum of the positive lift forces is equal to the sum of the negative lift forces. From the angle of zero lift, the curve is a straight line up to, in this case, about 12° angle of attack. Over this range C_L is directly proportional to the absolute angle of attack. Beyond this point C_L increases more slowly with increase of α, until at a certain angle, known as the *stalling angle* or *critical angle*, C_L reaches a maximum value, denoted by $C_{L_{max.}}$. In the case considered, the stalling angle is 20° and $C_{L_{max.}}$ is 1·46. The portion of the curve beyond the stalling angle is not of immediate interest, and we need note only that a large decrease in C_L occurs at this point.

$C_D \sim \alpha$ Curve

The minimum value of C_D occurs at a small angle of attack, in this case −1°. For symmetrical aerofoils it occurs at the angle of zero lift (0°), and the drag curve is then symmetrical about the ordinate through this angle. For unsymmetrical aerofoils the curve generally rises more steeply on the negative side of $C_{D_{min.}}$. For positive increases in α, C_D increases from $C_{D_{min.}}$ slowly at first then,

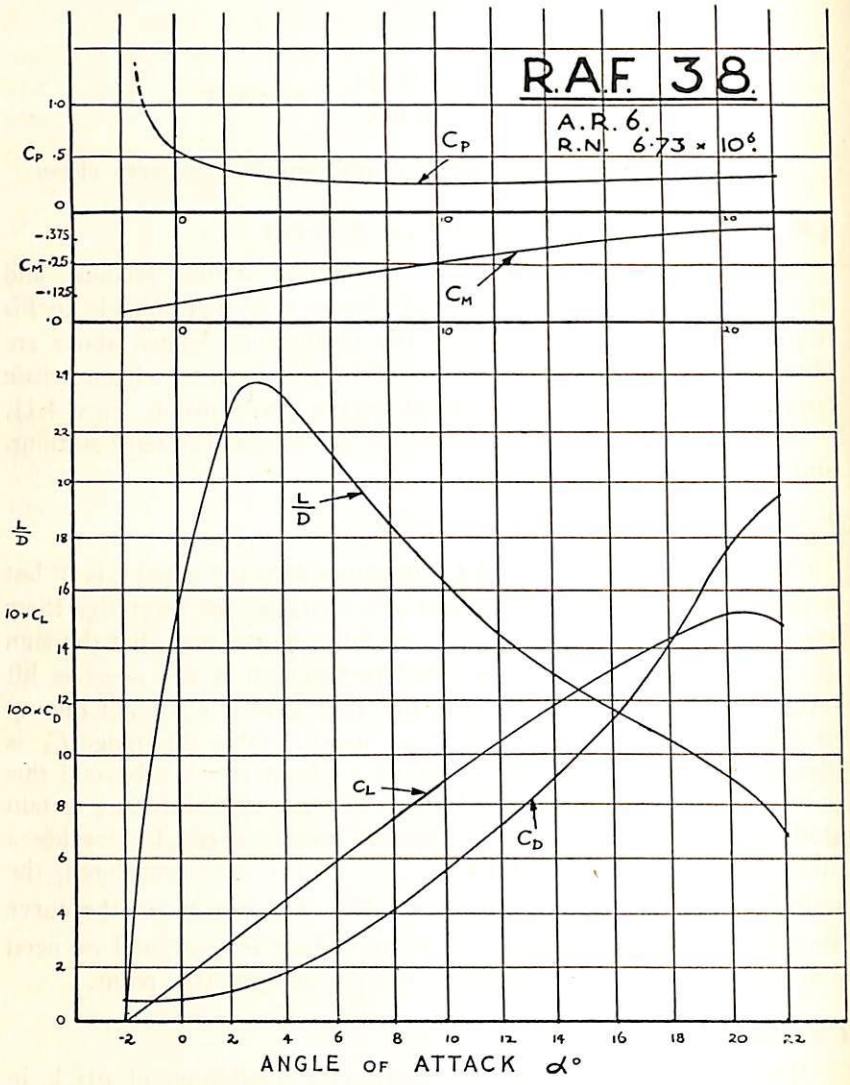


FIG. 4.11.—CHARACTERISTIC CURVES FOR AEROFOIL SECTION R.A.F.38.
(*Note :* R.N. = Reynold's Number. See para. 5.4.)

near the stalling angle, very steeply, the curve resembling a parabola in part.

$C_M \sim \alpha$ Curve

C_M varies almost linearly with α over the whole range of angles normally used. It should be noted that C_M is not zero at the angle of zero lift. This is explained by the fact that the positive and negative lift forces form a couple at this angle, so that there is no resultant lift but there is moment. Note that C_M is negative. The sign convention adopted is that a moment tending to raise the leading edge of the aerofoil is positive

$C_{C.P.} \sim \alpha$ Curve

At the angle of zero lift the centre of pressure is at an infinitely great distance behind the trailing edge. As α increases the C.P. moves forward until it reaches its foremost position near the stalling angle, beyond which point it moves backward again. A C.P. movement of this nature is termed "unstable," since, if an aerofoil is balanced at a particular value of α and α is suddenly increased, the forward shift of the C.P. and the increase in lift set up a moment which tends to increase α still further. This tendency continues, until eventually the aerofoil stalls. It may be noted that a flat plate has a "stable" C.P. movement.

A large variation of C.P. position is obviously undesirable for the wing of an aeroplane, and attempts have been made in the past to evolve aerofoils with little or no C.P. travel. Apart from symmetrical aerofoils, it is known that aerofoils with a "reflexed" or upturned trailing edge have a very small C.P. travel.

$\frac{L}{D} \sim \alpha$ Curve

The ratio $\frac{L}{D}\left(\text{or } \frac{C_L}{C_D}\right)$ may be used as one measure of the relative efficiency of various aerofoils, since it is the lift of the wing which sustains the weight of an aeroplane, whilst the drag furnishes part of the total resistance to forward motion. Consequently, it is desirable that a wing should provide a large lift and at the same time offer minimum resistance to forward motion. It should be noted that a high value of *lift–drag ratio*, as it is termed, is not necessarily the deciding factor in choosing an aerofoil. Other factors, which will become apparent later, must be taken into account.

Examination of the curve shown, which is of similar form for most aerofoils, shows that the maximum value of L/D occurs at an angle slightly greater than that corresponding to $C_{D_{min.}}$. A fuller discussion of the L/D ratio will be found later in the course.

Alternative Forms of the Characteristic Curves

It is generally more convenient to compare aerofoil sections at the same value of C_L, rather than at the same value of α. For this reason the curves are often drawn showing the other coefficients and α plotted against C_L. Sometimes, in order to obtain a straight line graph, C_D is plotted against C_L^2 and $C_{C.P.}$ against $1/C_L$. The curve of C_D against C_L is referred to as the *polar curve* of the aerofoil.

4·9. Pressure Distribution over an Aerofoil Section

In para 4.1 it is stated that the changes in pressure distribution resulting from the flow of air past a solid body result in an aerodynamic force on the body, and it is of interest to consider the lift and drag of an aerofoil section in relation to the distribution of pressure over the surface. The variation of the normal pressure over the surface of an aerofoil section is usually determined experi-

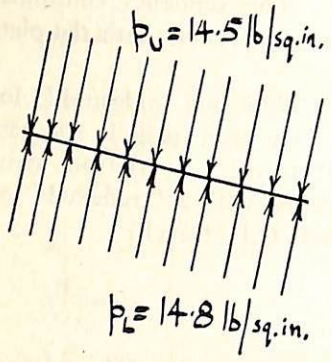


Fig. 4.12.

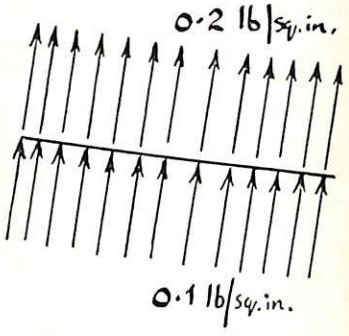


Fig. 4.13.

Pressure on Flat Plate.

mentally in a wind tunnel, as described in Chapter V. It is customary in such experiments to measure the difference between the normal pressure at various points on the section and the static pressure of the undisturbed stream, and it is found that, for positive angles of attack, the pressure over most of the upper surface is less than the static pressure of the undisturbed stream, whilst over the lower surface the normal pressure is generally greater than static.

The nett result is an upwards force on the section. That the difference between the pressure on the upper surface (p_U) and that on the lower surface (p_L) need not be large to provide an upwards force sufficient to lift an aeroplane may be understood if, for example, we consider a flat plate as in Fig. 4.12, with p_U assumed to be constant at 14·5 lb. per sq. in., and p_L assumed to have the constant value of 14·8 lb. per sq. in. The nett upwards pressure on the plate is 0·3 lb. per sq. in. If the total area of the plate is 500 sq. ft., the upwards force will be $500 \times 144 \times 0{\cdot}3$ lb., i.e., 21,600 lb., over $9\frac{1}{2}$ tons. If, instead of quoting actual pressures, as in Fig. 4.12, we quote the difference between the normal pressure and static pressure, assumed to be 14·7 lb. per sq. in., we obtain the distribution in Fig. 4.13, the arrows on the upper surface now pointing away from the surface since $(p_U - 14{\cdot}7)$ is negative. This method of representing pressure is seen to yield the same results as using the actual pressures. Adopting this method, the pressure distribution over an aerofoil section for several angles of attack is shown in Chapter V. It will be observed that the upper surface, generally, makes a greater contribution to the upwards force than the lower surface.

4.10. Stalling of Aerofoils

The stalling angle of an aerofoil has been mentioned in the section of para. 4.8 dealing with the $C_L \sim \alpha$ curve, and although a detailed discussion of the phenomenon termed " stalling " must be deferred, it is possible to give an elementary explanation of what happens when an aerofoil reaches its critical angle.

Whilst the angle of attack remains low, the airstream has no difficulty in following the upper surface of the aerofoil, but as the angle of attack is increased, the air flowing over the upper surface has to follow a curve which becomes much sharper. Eventually this curve becomes too sharp and the airstream breaks away from the surface of the aerofoil, leaving a region of violently eddying flow. The result is that the pressure over the upper surface increases, becoming more nearly equal to the static pressure of the undisturbed stream, so that a large amount of the upwards lifting force due to the negative pressure difference on the upper surface is lost. At the same time the " wake " of eddying flow is widened considerably and there is a large increase in drag. The flow past a stalled aerofoil is illustrated in Fig. 4.14.

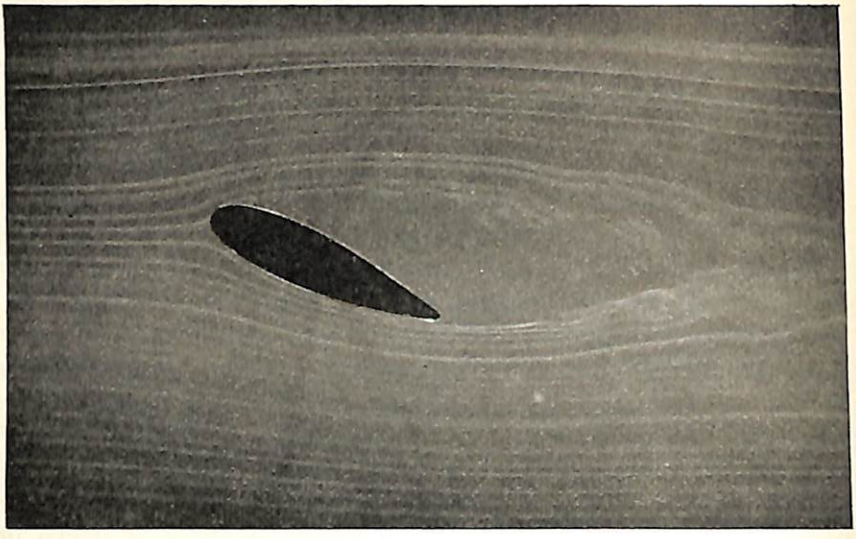


FIG. 4.14.—FLOW PAST A "STALLED" AEROFOIL.

Examples, Chapter IV

1. Describe the variation of C_L with α for a typical aerofoil, and explain the following terms :—Stalling Angle, No-Lift Line or Zero Lift Line, Absolute Angle of Attack.

2. Discuss the movement of the centre of pressure of an aerofoil as the angle of attack changes.

3. An aerofoil at 6° angle of attack has $C_L = 0{\cdot}866$, $C_D = 0{\cdot}0574$, $C_{M_{L.E.}} = -0{\cdot}2618$. Determine the centre of pressure coefficient by exact and approximate methods.

4. What is the percentage error in the centre of pressure position as determined by the approximate method for an aerofoil at 10° angle of attack with $C_L = 0{\cdot}92$, $C_D = 0{\cdot}047$ and $C_{M_{L.E.}} = -0{\cdot}284$.

5. Use the curves given in Fig. 4.11 to determine the angle of attack at which an aeroplane of 2000 lb. weight should fly at a speed of 120 m.p.h., if the wing area is 180 sq. ft. and the wing section is R.A.F. 38. What is the corresponding drag coefficient ?

6. What is the lift of a wing of R.A.F. 38 section, 400 sq. ft. in area, at 4° angle of attack and a speed of 130 ft. per sec. ? What is the drag ?

CHAPTER V

WIND TUNNELS

5.1. General

In Chapter IV we have discussed the characteristic curves of an aerofoil section. Such curves are obtained from measurements made in wind tunnels, and it is the purpose of this chapter to give a brief indication of the manner in which such tests are carried out. It will be necessary, first, to consider the layout and working of wind tunnels, of which there are several types.

The function of the wind tunnel is to enable measurements to be made of the forces acting on a body placed in a stream of air of known velocity. It is possible to carry out experiments for other purposes, such as investigating the flow round the body, or determining the distribution of pressure over the surface of the body under test.

5.2. Open Circuit Tunnel

This type of tunnel is illustrated in Fig. 5.1. Air is drawn in from the room through the intake, A, by means of the fan, F. The air passes through the honeycomb, B, designed to straighten out the airflow, to the working section, C, the tunnel generally contracting in cross-section from the intake to a point shortly before the working section. The tunnel expands in the regenerative cone, D, beyond the working section and, on passing the fan, is diffused into the surrounding room. Suitable balances are provided, from which the model may be supported in the working section, so that the forces acting on it may be measured.

This type of tunnel gives quite good results for a small tunnel in a comparatively large room. The smoke tunnel photographs in this book were obtained in a tunnel of this type, designed and built purely as a smoke tunnel at Loughborough College.

5.3. Return Circuit Tunnel

The majority of tunnels in use to-day for commercial testing are of the return circuit type, illustrated diagrammatically in Fig. 5.2. In this type the air, instead of being diffused into the room on passing

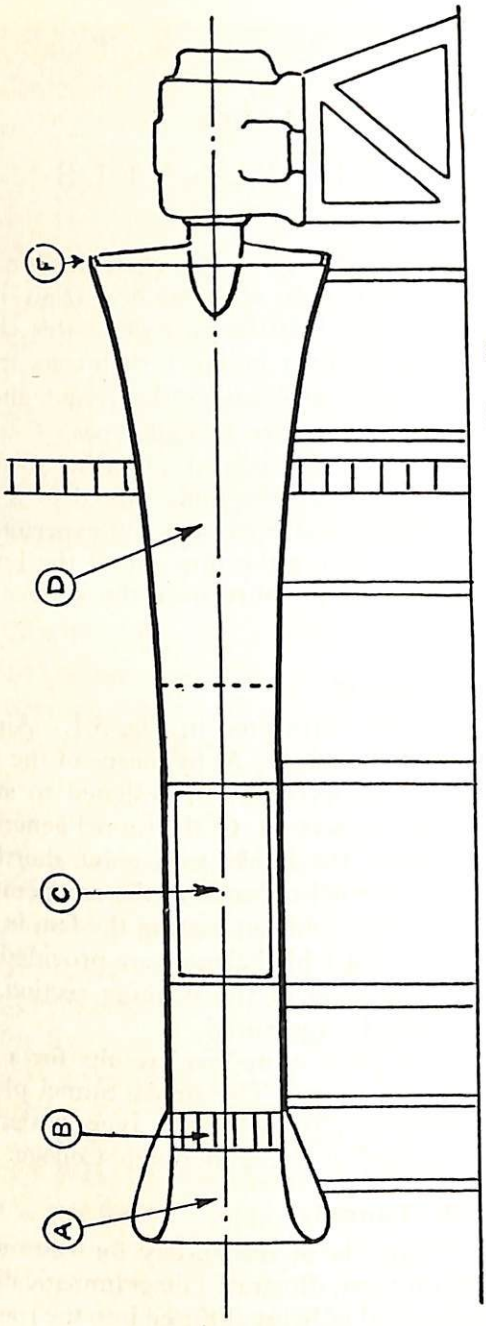


FIG. 5.1.—DIAGRAMMATIC ILLUSTRATION OF OPEN CIRCUIT WIND TUNNEL.

the fan, continues in a closed channel round to the working section again. Less power is needed for this tunnel than for the open circuit tunnel for the same speed, since the air is not continually being accelerated from rest. The air is guided round the corners by "cascades" of vanes, which are of aerofoil section designed to deflect the airstream through the required angle. It will be noted

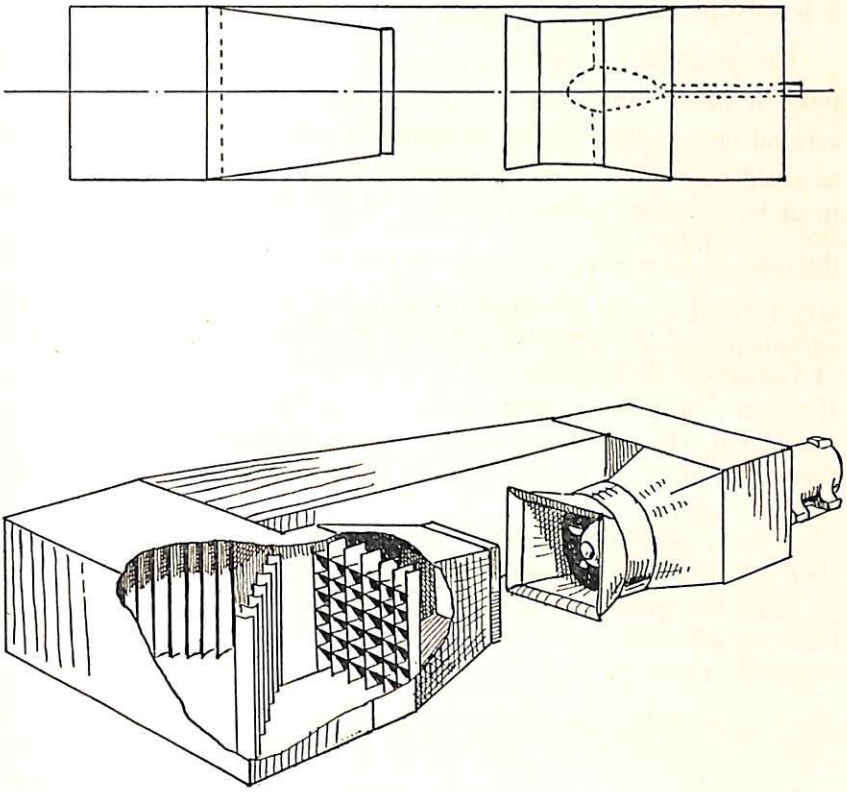


FIG. 5.2.—RETURN CIRCUIT WIND TUNNEL.

that the tunnel diverges from the outlet side of working section round to a point just before the entry to the working section, and then contracts suddenly. This contraction has the effect of increasing the airspeed at the working section over that elsewhere in the tunnel, and of steadying the flow. The 24-ft. diameter tunnel at the Royal Aircraft Establishment and the N.A.C.A. full-scale tunnel in the U.S.A. are return circuit tunnels.

In both types mentioned above, the working section may be opened or closed. In the latter case the walls of the tunnel are continuous across the working section, and in the former case the air is made to spring across a gap. The open section has the advantage of ease of accessibility, and results in a saving of material in the construction of the tunnel.

5.4. Compressed Air Tunnels

Except under certain conditions, the results obtained from model tests in wind tunnels cannot be applied directly to the full-scale aerofoil or aeroplane. One condition which must be met in order to avoid *scale effect* is that a quantity known as Reynold's Number must be the same for both model and full-scale. This quantity is the ratio $\rho\frac{VL}{\mu}$, where ρ denotes the density of the fluid (in this case air), V is the speed, L is a linear dimension of the model or full-scale aeroplane, usually in the direction of flow, and μ is the " coefficient of viscosity " of the fluid. A fuller explanation of scale effect and Reynold's Number will be given in a later chapter, but for the moment it will suffice to consider the implications of the necessity that this ratio shall be the same in the model tests as for the full-scale aeroplane.

If air is used at standard sea-level conditions in the wind tunnel, then ρ/μ will be the same in the two cases so that we should need to make the product VL the same. Hence, for a $\frac{1}{10}$ scale model of an aeroplane designed to fly at 200 m.p.h., the tunnel speed (V_m) required would be given by :—

$$V_m \times \frac{1}{10} L = 200 \times L$$

i.e.,

$$V_m = \mathbf{2000\ m.p.h.}$$

It is obvious that such a speed would certainly require prohibitive power. Consequently, other means of varying Reynold's Number must be found. One means, which is not very convenient, is to carry out the model tests in some fluid other than air. The method most commonly used is to increase the ρ/μ ratio of the air by compression, so that with a reasonable speed it is possible to attain Reynold's Numbers comparable with those in flight. Compression of the air has very little effect on μ, provided there is no great change

in temperature, so that using compressed air at 20 atmospheres, the speed required for the model referred to above, would be given by :—

$$\rho_m \frac{V_m L_m}{\mu_m} = \rho \frac{VL}{\mu}$$

i.e.,
$$\frac{20}{\mu} \rho V_m \frac{1}{10} L = \rho \frac{VL}{\mu}$$

$$V_m = \frac{V}{2} = \mathbf{100\ m.p.h.}$$

A tunnel which is used under these conditions is known as a Compressed-Air Tunnel, or as a Variable-Density Tunnel.

5.5. High-Speed Tunnels

Several types of high-speed tunnels exist, some designed to work at speeds in excess of the speed of sound. Of the subsonic tunnels, the best known in this country is that at the R.A.E., in which speeds around 600 m.p.h. are obtained without the use of excessive power by the expedient of lowering the density of the air in the tunnel.

5.6. Measurement of Forces on Models

The measurement of forces on models in wind tunnel tests are made by means of " wind tunnel balances," of which there are many types.

A simple balance for the measurement of lift or drag is shown in Fig. 5.3. The balance is mounted beneath the working section ; for the measurement of drag the balance arm is aligned with the axis of the tunnel, and for the measurement of lift it is placed at right angles to the axis.

The balance consists, essentially, of a lever arm AOB supported on a fulcrum O. The model M is carried on a vertical rod OC attached to the lever arm at O. The arm

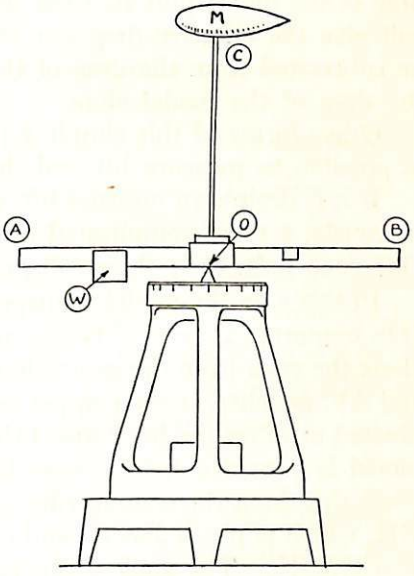


FIG. 5.3.—SIMPLE DRAG BALANCE.

AO is graduated and carries a rider weight, W. The arm OB carries another rider to serve as a counterweight. Before the tunnel is started, the weight W is set at a convenient zero and balance is obtained by means of the counterweight. When the tunnel is running, the drag force on the model will produce a moment about O, causing AOB to tilt. Balance is achieved by adjusting the position of W. Then, if the weight W has been moved a distance x along OA, the moment of the drag force about O must be equal to Wx. If the height of the centre of resistance of the model above O is known, it is possible to calculate the drag, since Wx is equal to the drag multiplied by the height.

To measure lift, the balance is swung round through 90 degrees, so that the arm AOB is perpendicular to the wind direction. In this position the balance will indicate the force on the model at right angles to the airstream, i.e., the lift.

In general, it will be necessary to correct the measured drag for the drag of the supporting rod and for drag due to interference between the rod and the model. The drag of the rod itself is easy to determine, but the evaluation of the interference drag presents difficulties. One method sometimes adopted is to use two supports, R_1 and R_2, each of which can be removed in turn. Subtracting the drag of the model plus R_2 from that of the model plus R_1 plus R_2, will give the effective drag due to the presence of R_1. This can be subtracted from the drag of the model plus R_1 to give, finally, the drag of the model alone.

Other forms of this simple type of balance exist with which it is possible to measure lift and drag simultaneously.

If it is desired to measure lift, drag and pitching moment simultaneously, a more complicated form of balance becomes necessary. One such balance is shown diagrammatically in Fig. 5.4.

In this case the model is suspended upside down in the tunnel. It is supported at each of two points (A in the diagram), equidistant along the span from the centre-line, by means of two fine wires AE and AF, attached at their upper ends to the trapeze GEF, which is pivoted at G on the lever arm GNW_2. Additional support for the model is provided by the wire CH, attached at C to a "sting" projecting from the trailing edge of the model, and at H to the arm GH, which is pivoted at G, and attached by a wire JK to the lever arm KMW_3. The angle θ can be varied at will, so that the angle of attack of the model may be adjusted. A horizontal wire EQ

extends from the trapeze to the cranked lever QPOW_1. The lines GA and HC must be vertical.

The drag is transmitted to the trapeze via AE and AF and thence to EQ, and is balanced by movement of W_1, so that the moment of the drag about O is known from the reading of the balance POW_1.

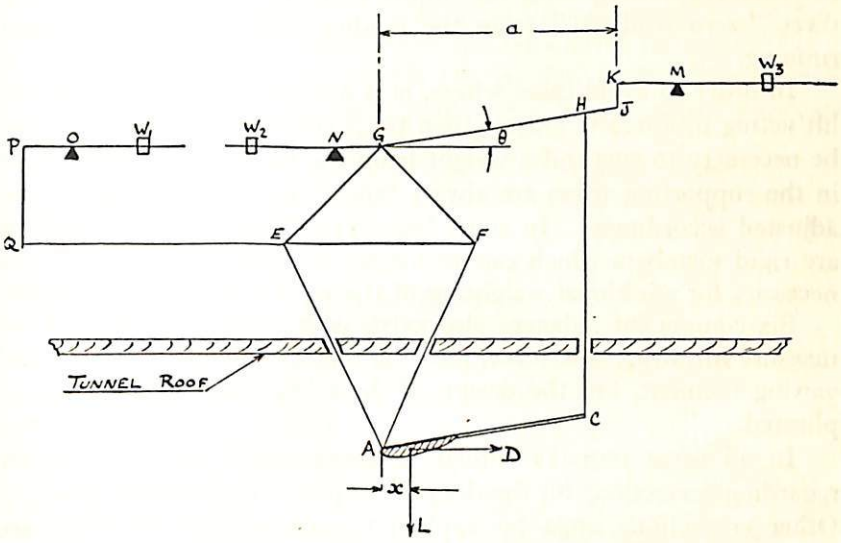


FIG. 5.4.—THREE COMPONENT BALANCE.

The whole of the lift is supported at G and H, so that we have

$$\text{Lift} = \frac{\text{Moment of } W_2 \text{ about N}}{\text{GN}} + \frac{\text{Moment of } W_3 \text{ about M}}{\text{KM}}$$

To determine the pitching moment, we note that the moment about A of the aerodynamic force on the model is equal to the moment about A of the force in CH. This in turn is equal to the moment about G of the force in JK. Hence, we have finally,

$$\text{Pitching moment about A} = \frac{\text{Moment of } W_3 \text{ about M}}{\text{KM}} \times a$$

If, as an approximation, we assume that the line of action of the drag (D) passes through A, then the pitching moment about A is equal to the lift (L) multiplied by x, which is the distance of

the centre of pressure from A measured parallel to the airstream.

Hence, $$x = \frac{\text{Moment of } W_3 \text{ about M}}{\text{KM}} \times \frac{a}{L}$$

It will be observed that the above takes no account of the weight of the model. The necessary corrections are obtained by adjusting the balances initially with the tunnel switched off, and subtracting these "zero readings" from the readings taken with the tunnel running.

In order to avoid cases where, at a negative angle of attack, the lift acting upwards is greater than the weight of the model, it may be necessary to suspend a weight from the model so that the loads in the supporting wires are always tensile, the zero readings being adjusted accordingly. In some balances of this type the supports are rigid members which can withstand compressive loads and the necessity for additional weighting of the model does not then arise.

Six-component balances also exist, with which it is possible to measure lift, drag, side-force, pitching moment, rolling moment and yawing moment, but the design of these becomes somewhat complicated.

In all cases remarks similar to those made previously apply regarding corrections for the drag of supports and interference drag. Other corrections must be applied to take account of what are broadly termed "tunnel effects," and these will be dealt with later.

5·7. Measurement of Wind Speed

An obvious means of measuring the wind speed in a tunnel is to use a pitot-static tube, placed in the stream at the working section, in conjunction with a manometer, but since such an arrangement would interfere with the location of models for other experiments an alternative method must be employed. A small hole is drilled in the tunnel wall upstream of the working section and a connection is taken from this hole to one side of a manometer, the other side of which is open to atmospheric pressure. The readings of this manometer are calibrated throughout the speed range against $\frac{1}{2}\rho V^2$ evaluated from the readings of another manometer connected to a pitot-static tube placed in the working section. Once this calibration has been performed, the value of $\frac{1}{2}\rho V^2$ in the working section is known without the necessity of using a pitot-static tube.

5.8. Pressure Plotting Experiments

A knowledge of the distribution of pressure around an aerofoil section is necessary for the design of the internal structure of an aeroplane wing, and it is customary to determine the pressure distribution experimentally in a wind tunnel. A typical experimental set-up is illustrated in Fig. 5.5. At mid-span of the model aerofoil there is a series of small holes in the surface of the aerofoil, each of which communicates with the outside of the tunnel via a small bore tube contained within the aerofoil itself. These tubes are connected to a multi-tube manometer, the common reservoir

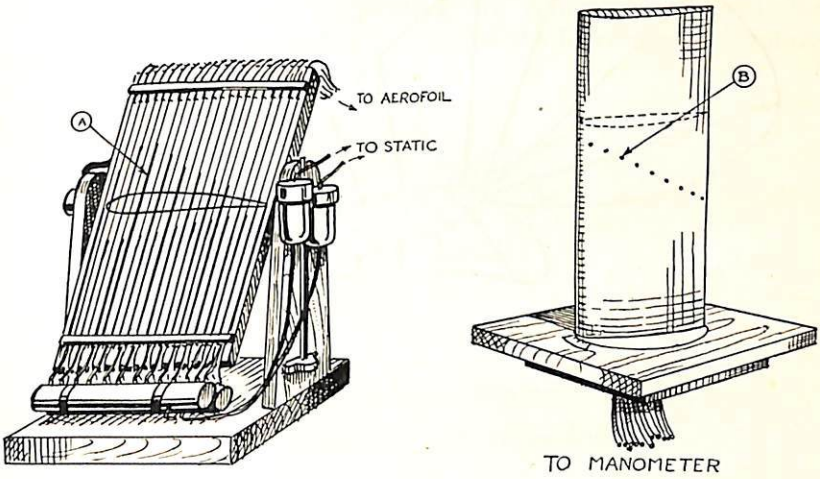


FIG. 5.5.—MULTI-TUBE MANOMETER AND AEROFOIL FOR PRESSURE DISTRIBUTION EXPERIMENTS.

of which is connected to the static pressure hole in the wall of the tunnel. On the particular manometer shown in the figure the outer glass tube on each side is connected to the tunnel static pressure hole at its upper end, so that the level of the liquid in these tubes serves as a datum from which the level in the other tubes can be measured. Considering any tube, A, the pressure applied at its upper end is the pressure normal to the aerofoil surface at the corresponding hole, B. The difference in height between the level of the liquid in A and the datum line will give the pressure head corresponding to the difference between p, the normal pressure at B, and p_0, the static pressure in the tunnel. One tube of the manometer

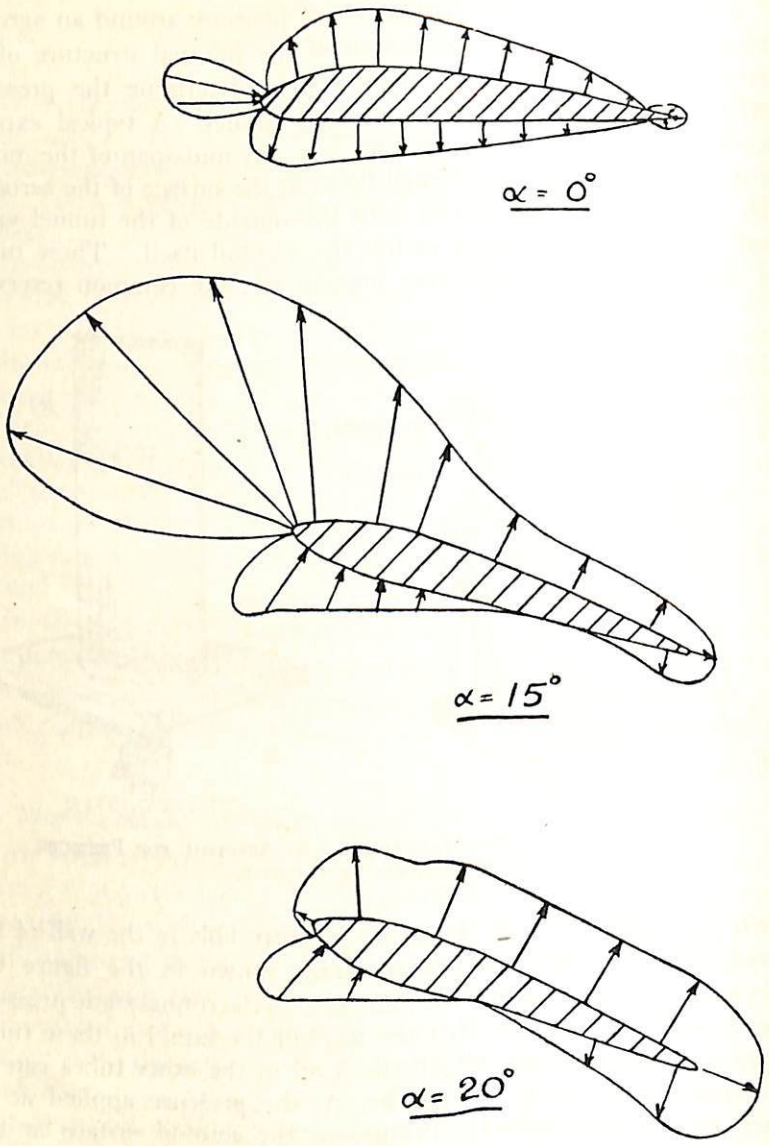


FIG. 5.6.—PRESSURE DISTRIBUTION ROUND AN AEROFOIL SECTION.

may be left open to atmospheric pressure at its upper end, the difference in height between the level of the liquid in this tube and the datum line being proportional to $\frac{1}{2}\rho V^2$. It is possible, therefore, from the readings of this manometer to obtain the ratio $(p - p_0)/\frac{1}{2}\rho V^2$ for each hole in the aerofoil surface. The actual pressure difference, $p - p_0$, in lb. per sq. ft., may be evaluated quite simply by making use of the calibration of the tunnel wall manometer described in para. 5.7 to determine the value of $\frac{1}{2}\rho V^2$. For convenience it is usual to plot the ratio $(p - p_0)/\frac{1}{2}\rho V^2$, and typical results of experiments with the above apparatus are depicted in Fig. 5.6, where this ratio is plotted normal to the surface.

Two other methods of plotting are commonly adopted, the value of which will be made apparent by the following explanation.

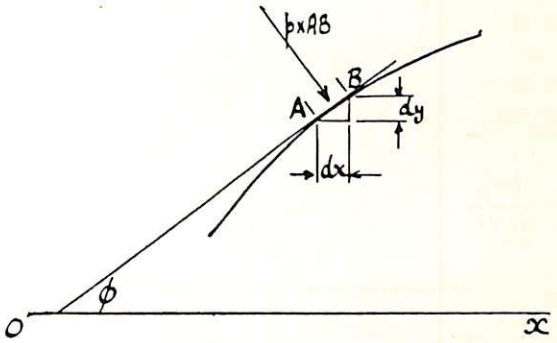


FIG. 5.7.—RESOLUTION OF PRESSURE FORCE.

Consider (Fig. 5.7) the normal force on an element AB of the aerofoil contour, of unit width perpendicular to the plane of the paper. The tangent to AB makes an angle ϕ with the axis Ox as shown, and the projection of AB on the x axis is dx. If the normal pressure on AB is p, the normal force on AB is $p.\mathrm{AB}.1$. The component of this force perpendicular to Ox is $p.\mathrm{AB}.\cos\phi$, which is equal to $p.dx$. This applies to all elements of the contour, so that the total force per unit span perpendicular to Ox is $\Sigma p dx$. Since the static pressure acts equally in all directions, instead of p we need consider only the difference between p and the static pressure, p_0. Hence the total force perpendicular to Ox is $\Sigma(p - p_0)dx$ per unit span. It follows that if the aerofoil is projected on to the line of flight, which may be regarded as our x axis, and if the pressure difference is plotted on this axis as base, the area under the curve so obtained

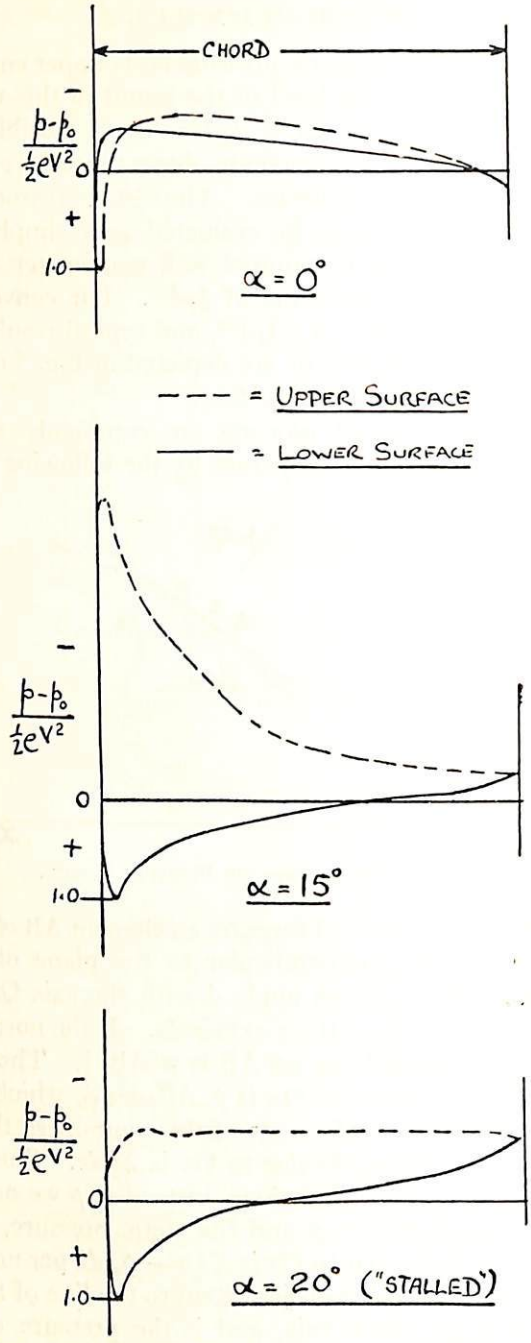


FIG. 5.8.—PRESSURE RATIO PLOTTED AGAINST CHORD.

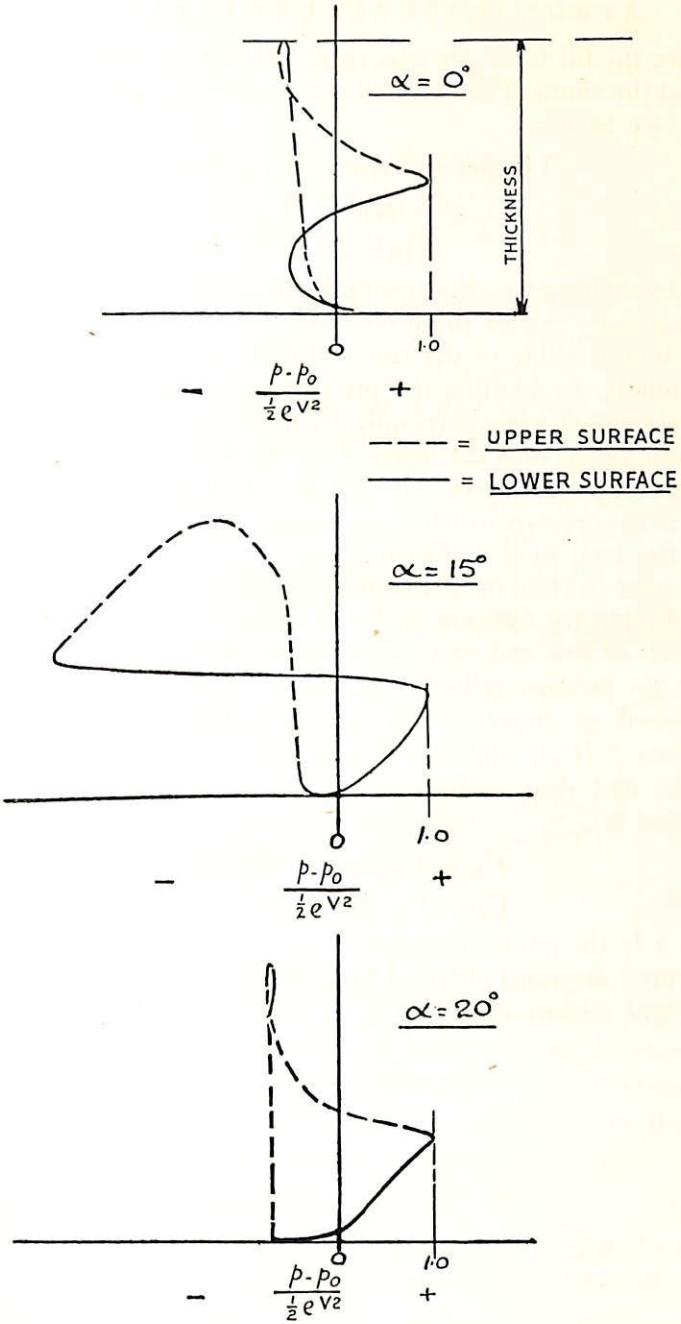


FIG. 5.9.—PRESSURE RATIO PLOTTED AGAINST AEROFOIL THICKNESS.

will give the lift force per unit span. If the local lift coefficient is C_L, and the chord of the aerofoil is c, so that the area per unit span is $c.1$, we have :—

$$\text{Lift per unit span} = C_L.c.\tfrac{1}{2}\rho V^2$$

i.e., $$C_L = \Sigma \frac{(p - p_0)dx}{c.\frac{1}{2}\rho V^2} = \frac{1}{c}\Sigma \frac{(p - p_0)}{\frac{1}{2}\rho V^2}dx$$

hence by dividing by c the area under the curve obtained by plotting $(p - p_0)/\frac{1}{2}\rho V^2$ against the projection of the aerofoil on the line of flight, it is possible to determine the lift coefficient, C_L.

Similarly, by plotting the pressure ratio against the projection of the aerofoil on a line perpendicular to the line of flight it is possible to determine the drag coefficient, C_D. It should be noted, however, that this method will give only that part of the drag coefficient which is due to the pressure distribution. Other factors yet to be discussed affect the total drag coefficient.

In order to avoid the necessity of projecting on to several different lines of flight for different angles of attack, it is usual to project on to the chord line and on to a perpendicular to the chord line, and to plot the pressure ratio on these lines. Force coefficients may be determined as above, which can be called the "normal force coefficient" (C_N), and the "tangential force coefficient" (C_T). The lift and drag coefficients may then be obtained by simple resolution as

$$C_L = C_N \cos\alpha - C_T \sin\alpha$$
$$C_D = C_N \sin\alpha + C_T \cos\alpha$$

where α is the angle of attack.

Typical diagrams obtained by plotting the pressure ratio against chord and thickness are shown in Figs. 5.8 and 5.9.

CHAPTER VI

ELEMENTARY PRINCIPLES OF FLIGHT

6.1. The Loads on an Aeroplane in Steady Flight

THE loads acting on an aeroplane in steady, horizontal flight are shown in Fig. 6.1. These forces are as follows:—

W is the weight of the aeroplane acting through the C.G.

L ,, ,, lift of the wings acting through the C.P.

D_w ,, ,, drag ,, ,, ,, ,,

D_p ,, ,, ,, of the remainder of the aeroplane (*parasite drag*) acting through the *centre of drag*.

T ,, ,, thrust of the propeller acting along the axis of the propeller.

P ,, ,, tail load acting through the C.P. of the tailplane.

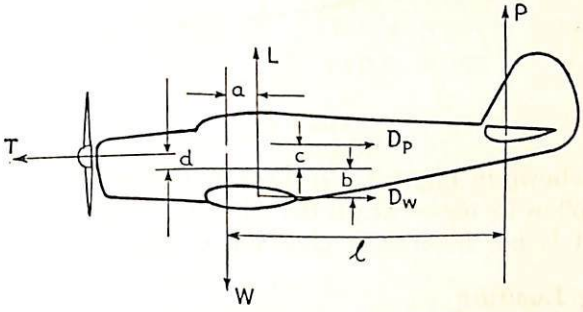


FIG. 6.1.—LOADS ON AN AEROPLANE IN STEADY HORIZONTAL FLIGHT.

For steady horizontal flight these forces are in equilibrium, and the conditions of static equilibrium may be used to determine the unknown quantities. However, certain simplifying assumptions enable us to make some preliminary observations.

6.2. Lift–Weight Relationship

In general the tail load, P, will be very small compared with the lift, L, and may be neglected. Consequently, we may write, approximately,

$$W = L = C_L \tfrac{1}{2}\rho S V^2 \quad . \quad . \quad . \quad \text{Eq. 6.1}$$

where C_L is the lift coefficient of the wings, S is the wing area, and V is the forward speed.

Since W is a constant, L must be constant for steady horizontal flight. Hence, the minimum speed at which horizontal flight is possible will be obtained when C_L has its maximum value. Denoting this minimum speed by V_s, we have

$$W = L = C_{L_{max.}} \tfrac{1}{2}\rho S V_s^2$$

or

$$V_s = \sqrt{\frac{W}{\tfrac{1}{2}\rho S . C_{L_{max.}}}} \quad . \quad . \quad \text{Eq. 6.2}$$

V_s is the *stalling speed* of the aeroplane.

In order to attain $C_{L_{max.}}$ the wing must be at a high angle of attack, so that the aeroplane will be flying horizontally in attitude

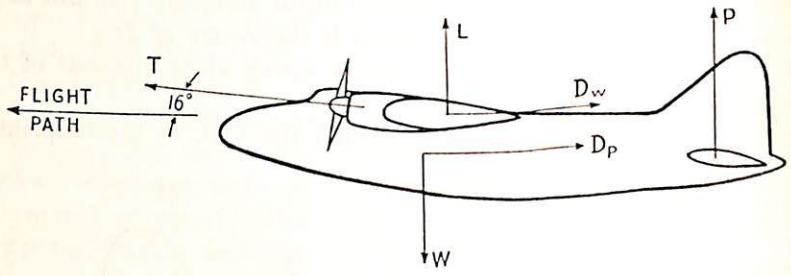


FIG. 6.2.

such as is shown in Fig. 6.2. It must be realised that an aeroplane does not follow its nose—i.e., if the nose of the aeroplane is pointing upwards it is not necessarily climbing.

6.3. Wing Loading

The ratio W/S is known as the wing loading, and is denoted by w. We may write

$$V_s = \sqrt{\frac{w}{\tfrac{1}{2}\rho C_{L_{max.}}}}$$

Consequently, a high wing loading leads to a high stalling speed, and since an aeroplane usually lands at an airspeed slightly above V_s, this means also a high landing speed. Since, in other circumstances, a high wing loading is often desirable, means have been adopted (slots and flaps—see later) for increasing the value of $C_{L_{max.}}$ at low speeds, so reducing V_s.

6.4. The Drag Equation

We may assume, approximately, that, in steady horizontal flight, the thrust, T, is equal to the total drag :—

i.e., $$T = D_T = D_w + D_p \quad . \quad . \quad . \quad \text{Eq. 6.3}$$

If we multiply the drag by the speed V (ft. per sec.) we obtain the work done per second in moving the aeroplane forward at this speed. Since one horsepower is equal to 550 ft. lb. per sec., the horsepower required to move the aeroplane forward at speed V is

$$\text{T.H.P. (thrust horsepower)} = \frac{TV}{550} = \frac{D_T V}{550} . \quad \text{Eq. 6.4}$$

It is usual to express the total drag of the aeroplane in the form of a drag coefficient, C_{D_T}, so that

$$D_T = C_{D_T} \tfrac{1}{2}\rho S V^2$$

Hence, $$\text{T.H.P.} = \frac{C_{D_T} \tfrac{1}{2}\rho S V^3}{550}$$

E.g., An aeroplane, weight 6000 lb., has an R.A.F. 38 section wing, 240 sq. ft. in area. What horsepower is required for the wing at a speed of 180 m.p.h. at sea level ?

$$C_L = \frac{W}{\tfrac{1}{2}\rho S V^2}$$

where $W = 6000$ lb.
$S = 240$ sq. ft.
$V = 264$ ft. per sec.
$\rho = 0{\cdot}00238$ slugs per cu. ft.

$$\therefore C_L = \frac{6000}{\tfrac{1}{2} \times 0{\cdot}00238 \times 240 \times (264)^2} = \mathbf{0{\cdot}3015}$$

From Fig. 4.11, when $C_L = 0{\cdot}3015$,

$$\frac{L}{D} = \frac{C_L}{C_D} = 23$$

$$\therefore \quad D = L \times \frac{D}{L} = \frac{W}{L/D} = \frac{6000}{23} = 261$$

$$\therefore \text{T.H.P.} = \frac{D \times V}{550} = \frac{261 \times 264}{550} = \mathbf{125}$$

6.5. Complete Balance of Forces

For consideration of stability, and to determine the tail load to be used for structural design purposes, it is necessary to solve the complete equations resulting from the conditions of equilibrium.

Referring to Fig. 6.1, these equations are :—

$$L + P - W = 0 \quad . \quad \text{Eq. 6.5}$$

$$D_w + D_p - T = 0 \quad . \quad \text{Eq. 6.6}$$

$$L.a + D_w b + P.l + T.c - D_p d = 0 \quad . \quad \text{Eq. 6.7}$$

The first two of these equations are obtained by resolving vertically and horizontally respectively. The third is obtained by taking moments about the C.G. of the aeroplane.

The solution of these equations requires a knowledge of the distances a, b, c, d and l, i.e., the positions of the C.P. of the wing, the centre of resistance of the remainder of the aeroplane, the C.P. of the tailplane (usually assumed to be at $\frac{1}{4}$ of the chord of the tailplane from the leading edge) and the thrust line relative to the C.G. must be determined.

The above assumes that the thrust line is horizontal, but it will be apparent that this is not necessarily the case ; if the thrust line is inclined to the horizontal, the vertical component of the thrust will appear in equation 6.5 and the horizontal component in equation 6.6.

Flight other than steady horizontal flight will be considered later, and it suffices to point out here that the lift, L, is not necessarily directed vertically upwards—it is perpendicular to the line of flight.

A numerical example follows which illustrates the above procedure.

Example. The aeroplane, shown in Fig. 6.2, is flying horizontally at a speed of 110 ft. per sec. The weight, W, is 8000 lb., and the thrust horse power is 664. The C.P. of the wing is 12 in. behind and 37 in. above the C.G., and the C.P. of the tailplane is 200 in. aft of the C.G., these distances being measured parallel and perpendicular to the line of flight. The thrust line is inclined upwards at 16° to the horizontal, and the perpendicular distance from the C.G. to the thrust line is 11 in. The parasitic drag is assumed to act through the C.G., and the parasitic drag coefficient, C_{D_p}, based on wing area, is 0·026. The wing area is 400 sq. ft. Determine the lift, wing drag and tail load.

The equations of static equilibrium are :—

$$L + P + T \sin 16° - W = 0 \quad . \quad . \quad (1)$$

$$D_w + D_p - T \cos 16° = 0 \quad . \quad . \quad (2)$$

$$L \times 12 - D_w \times 37 + P \times 200 + T \times 11 = 0 \quad . \quad . \quad (3)$$

From the data of the problem,

$$T = \frac{\text{T.H.P.} \times 550}{V} = \frac{664 \times 550}{110} = \mathbf{3320\ lb.}$$

$$\therefore\ T \sin 16° = 3320 \times 0{\cdot}2755 = \mathbf{915\ lb.}$$

and $$T \cos 16° = 3320 \times 0{\cdot}9612 = \mathbf{3190\ lb.}$$

$$D_p = C_{D_p} \times \tfrac{1}{2}\rho S V^2 = 0{\cdot}026 \times \tfrac{1}{2} \times 0{\cdot}00238 \times 400 \times (110)^2 = \mathbf{150\ lb.}$$

From equation 2 we have by substitution,

$$D_w = 3190 - 150 = \mathbf{3040\ lb.}$$

Equation 3 becomes :—

$$12L + 200P = 112{,}480 - 36{,}520 = 75{,}960 \quad . \quad . \quad (4)$$

Equation 1 becomes :—

$$L + P = 8000 - 915 = 7085 \quad . \quad . \quad . \quad . \quad (5)$$

From equations 4 and 5,

$$188P = 75{,}960 - 85{,}020 = -9060$$

$$P = \mathbf{-48\ lb.}$$

$$L = \mathbf{7085 + 48 = 7133\ lb.}$$

(NOTE : The negative value obtained for P indicates that the tail load acts downwards in this case, not upwards as was assumed in writing equations 1 and 3.)

Examples, Chapter VI

1. Calculate the stalling speed of an aeroplane given the following particulars : Weight = 1980 lb., wing area = 194 sq. ft., $C_{L_{max.}} = 2{\cdot}0$.

What will be the stalling speed with 200 lb. overload ?

2. For a particular aeroplane $C_L = 0{\cdot}07\alpha + 0{\cdot}1$, where α is the angle of attack in degrees.

If the weight of the aeroplane is 3000 lb., and the wing loading is 20 lb. per sq. ft., determine the speed of level flight corresponding to values of α of 0°, 2°, 4°, 6° and 8°.

3. If the landing speed of an aeroplane at sea level is 60 m.p.h., what would be the landing speed at an aerodrome which is 4000 ft. above sea level ?

4. An aeroplane weighing 6200 lb. has a wing of R.A.F. 38 section of 250 sq. ft. area. What horsepower is required to overcome the wing drag at a speed of 150 m.p.h. at sea level and at 10,000 ft. altitude?

5. An aeroplane with wing loading of 20 lb. per sq. ft. has a wing of R.A.F. 38 section and 300 sq. ft. area. Determine the angle of attack at speeds of 150 and 200 m.p.h., and calculate the horsepower required to overcome the wing drag at these speeds, all at sea level.

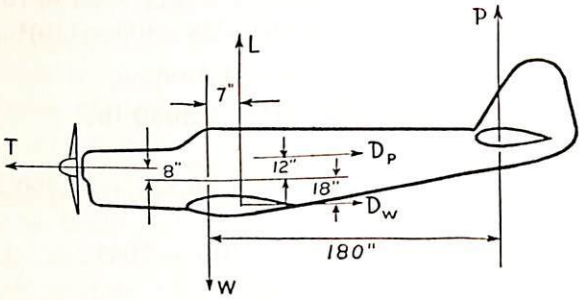


FIG. 6.3.

6. The aeroplane shown in Fig. 6.3 is flying horizontally at a speed of 130 m.p.h. The parasitic drag coefficient, based on wing area, is 0·024 and the wing area is 194 sq. ft. The weight of the aeroplane is 1980 lb. Calculate the lift, wing drag and tail load.

CHAPTER VII

GENERAL PRINCIPLES AND PROCEDURE IN AIRCRAFT DESIGN

7.1. General

THE design of an aeroplane resolves itself into two main sections—aerodynamic design and structural design—but it must be realised at the outset that neither section is completely independent of the other, and that it is often necessary to compromise between aerodynamic and structural requirements.

In this chapter the general principles of aeroplane design will be given in broad outline, the details being filled in later when a more advanced stage has been reached in the study of aerodynamics and structures.

7.2. General Design Requirements

The major items to be considered by the designer may be summarised as follows (note that the order given is not intended to be that of importance) :—

(1) Load required to be carried,
(2) Performance and other aerodynamic requirements,
(3) Type of material, structural and general arrangement,
(4) Weight,
(5) Power plant.

Before a design is commenced, a specification is drawn up giving the purpose for which the aeroplane is required, the performance to be provided, and often stating the type of aeroplane and the power plant to be used. Thus the specification may fix items (1) and (5) completely and (2) and (3) in part. Of the remaining items, that causing most trouble to designers is (4). The saving of weight is of paramount importance in aircraft design, and it is essential that the structure shall be as efficient as possible.

7.3. General Procedure

Once the specification has been received or drawn up, and the type of aeroplane decided upon, the design proceeds roughly as follows :—

1. AERODYNAMIC
   - (*a*) Preliminary weight estimate.
   - (*b*) Selection of aerofoil section for wing and determination of wing area and planform.
   - (*c*) Estimate of fuselage dimensions and general shape.
   - (*d*) Estimate of sizes of tail surfaces.
   - (*e*) Preliminary three-view drawing.
   - (*f*) Provisional performance check.
   - (*g*) Detailed weight estimate.
   - (*h*) Balance diagram and determination of C.G. position.
   - (*i*) Revision of wing area, etc.
   - (*j*) Final three-view drawing.
   - (*k*) Performance estimate and stability calculation.

2. STRUCTURAL
   - (*a*) Determination of design loads.
   - (*b*) Layout and stress analysis of structure.

The above steps are not independent, and several of them may proceed simultaneously. The list given is not complete, but represents a rough idea of the general procedure to be followed. The exact method of tackling each problem depends upon the personal ideas of the designer, and with experience it is possible to form a very rapid estimate of the weight of a particular aeroplane based simply on previous aeroplanes of similar types.

In the absence of information on the weights of previous aircraft of similar type, one method commonly adopted in forming a preliminary estimate of the weight, is to determine the non-structural weight—i.e., power plant, fuel and oil, passengers and crew, baggage, instruments, etc. For most conventional aeroplanes the structure weight is between 30% and 35% of the total weight, so that it is possible to assume the non-structure weight as between 65% and 70% of the total weight. With further advances in structural design it is possible that the structure weight percentage may be reduced, but as an initial estimate it is sufficient, in general, to use

the figures quoted. For example, if the non-structural weight of a particular aeroplane were determined as 7000 lb., we could make a preliminary estimate of the all-up weight as approximately 10,000 lb.

From a knowledge of the characteristics of the aerofoil section selected for the wing, an initial estimate of the wing area may be obtained as the area required to provide a lift equal to the weight at the proposed landing speed, using the maximum value of C_L—if it is proposed to fit high lift devices, such as flaps, for landing purposes, a value for $C_{L_{max.}}$ of about 2·0 may safely be assumed for conventional aerofoils.

The fuselage must be so designed as to provide sufficient room to carry the crew, passengers and baggage, together with any items of equipment which it is proposed to locate in the fuselage. In order to reduce drag to a minimum, a good streamlined shape is desirable, but structural difficulties must always be kept in mind.

The determination of the size of the tail surfaces is generally based on empirical formulæ which are results of past experience, and the general consideration of the factors affecting size of tail surfaces will be dealt with later in the course.

At this stage in the design it is possible to produce a three-view general arrangement drawing of the aeroplane, and to make a rough estimate of the drag so that a check may be made on the possibility of meeting the major performance requirements.

A detailed weight estimate follows, the various component parts being considered separately, and a *balance diagram* is prepared showing the location of the centre of gravity of the various items. From this information it is then possible to determine the position of the C.G. of the complete aeroplane, and to proceed to a complete estimate of the performance to be expected.

It is almost certain that several changes will be necessary before the weight and C.G. position are satisfactory, and it is essential that at all stages of the design close attention should be given to any possibility of weight saving.

The brevity of the above indication of the broad lines of procedure followed in the aerodynamic design of an aeroplane does not by any means imply that this branch of design is very simple, and requires only that the designer should follow certain empirical rules. The difficulties are many, as are the occasions on which compromise is necessary between what is desirable aerodynamically and what is necessary structurally.

In order to proceed to the structural design it is necessary to ascertain the loads which the various parts of the aeroplane will be required to withstand.

7·4. Load Factors and Factors of Safety

If, whilst an aeroplane is in steady horizontal flight at a speed V, the pilot manipulates the controls so that there is a sudden increase in the angle of attack of the wing without any immediate loss of speed, it is apparent that the lift of the wing will be increased so that it is greater than the weight of the aeroplane, and the forces acting are no longer in equilibrium. The effect of such a manœuvre is to produce an acceleration upwards and, since drag is also increased, backwards. Such acceleration must be taken into account in determining the loads to be withstood by the aeroplane. It is not possible to cater for the maximum possible acceleration which could be imposed on an aeroplane, and as the result of long experience the authorities in various countries have evolved requirements regarding acceleration to which aeroplanes must be designed. Suppose that the manœuvre is such as to make the lift n times its value in steady horizontal flight at the same speed. The aeroplane is then said to experience an acceleration normal to the flight path of $(n - 1)g$, where g is the acceleration due to gravity, and the *load factor* is said to be n. Since the lift in steady horizontal flight is equal to the weight, we may define *load factor* as *the ratio of lift to weight.*

The maximum normal accelerations specified are those which would not normally be exceeded in any manœuvre to which the aeroplane is likely to be subjected, but to allow for cases in which the pilot may inadvertently impose a greater acceleration on his machine, the *unit* design loads are multiplied by a so-called *factor of safety*, generally 1·5. This is not a factor of safety in the general engineering sense, since it cannot be asserted definitely that the design loads so obtained cannot be exceeded. For details of requirements relating to load factors, the student should refer to the "Airworthiness Requirements for Civil Aeroplanes" of the Air Registration Board.

7·5. Structural Design

No attempt will be made here to go into great detail regarding the structural design of an aeroplane. A brief indication will be given of the basic principles involved, and the subsequent chapters

of this volume will be devoted to a simple presentation of some of the essential parts of the theory of materials and structures.

An aeroplane structure is made up of beams, which are subjected to bending actions ; struts which carry compressive forces ; torsion tubes, subjected to twisting ; and many components which combine the functions of these. Some parts of the structure may be regarded as complete frameworks such as, for example, many fuselages, which

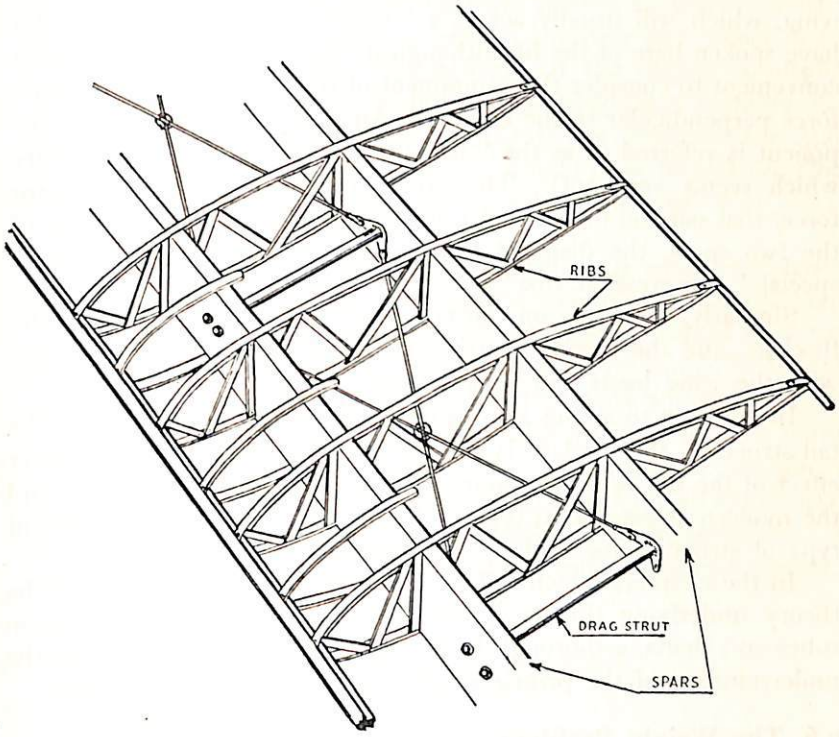


FIG. 7.1.—WING STRUCTURE.

carry bending loads by means of compressive and tensile end loads in the various members composing them. A particular example of a beam is the mainplane. Let us consider, for a moment, a typical wing structure, and see how it deals with the various forces acting on it. For simplicity we will take the case of a fabric-covered, two-spar monoplane wing, so that we can assume that the covering contributes nothing to the strength. We note in Fig. 7.1 that the structure consists essentially of two spars and a number of ribs

joining them, with some diagonal bracing. The ribs have two main purposes ; to maintain the contour of the wing to give it the correct aerofoil section, and to transmit the air loads to the spars. The proportion of the load on each rib which is applied to the spars depends on the position of the centre of pressure of the lift. The spars, loaded by the ribs, act as beams and transmit the loads to the fuselage joints. It should be noted that, in addition to the aerodynamic loads on the wing, the spars carry also the weight of the wing, which will usually act as a " relief " to the air loads. We have spoken here of the lift although it would, in general, be more convenient to consider the component of the resultant aerodynamic force perpendicular to the chord (in American textbooks this component is referred to as the " beam component "—a nomenclature which seems very apt). The other component of the resultant force, that parallel to the chord, is taken by the frame composed of the two spars, the diagonal bracing, and the ribs (or sometimes special " compression ribs ").

Similarly, the loads on the tail structure are transferred to the fuselage, and the fuselage carries those loads along to balance out with the wing loads and weight loads.

In addition to acting as a beam in carrying the loads from the tail structure, the fuselage is also subjected to twisting by the lever effect of the fin, or by unsymmetrical loading on the tailplane, and the modern stressed-skin type of fuselage provides a very efficient type of structure for dealing with this combination of loads.

In the chapters which follow will be found the elements of the theory underlying the design of frameworks, beams and torsion tubes and shafts, a thorough knowledge of which is essential to the understanding of the principles of structural design of aircraft.

7.6. The Weight Problem

Weight, in particular structure weight, plays such an important part in the design of an aeroplane that a brief discussion of the weight problem is not out of place.

When an aeroplane is designed, the performance is estimated using an assumed all-up weight. Since the non-structural load will usually be fixed by the specification, an accurate estimate of the all-up weight depends on the accuracy of the estimation of the structure weight. Let us suppose that a designer has decided that the all-up weight of his aeroplane will be W_1, of which W_s is

structural and W_n is non-structural, and let us consider what the effect would be if, when the aeroplane is built, the structure weight is W_s', greater than W_s, so that the total weight, W_2, is now $W_n + W_s'$. For a weight W_1, the relationship,

$$W_1 = L_1 = C_{L_1} \tfrac{1}{2} \rho S V^2$$

will hold for a speed V in steady horizontal flight. But, with weight W_2, if we are to maintain the same speed V, obviously C_L must be greater than C_{L_1}, which will, in turn, involve an increased value of C_D, an increased thrust to overcome the drag and, hence, more power from the engine. Alternatively, if the engine is already developing maximum power under these conditions of flight, the speed must be reduced, so that the thrust can overcome the drag. If the performance is not to suffer, the weight must be reduced from W_2 to W_1, a procedure which will involve reduction of W_n—i.e., a reduction in payload.

The unfortunate effects of an excess structure weight emphasize the necessity for a detailed, and accurate, weight estimate, and point further to the need for strict control to be maintained at all stages of the structural design. No part of the structure should be heavier than is necessary to provide the required strength and rigidity. Of course, there are unavoidable instances when a component which carries little load would, if designed to that load, be too flimsy. In such cases compromise is necessary but, in general, all unnecessary weight must be eliminated from the structure of an aeroplane. It should be remembered that "a pound saved is a pound gained."

CHAPTER VIII

FRAMEWORKS

8.1. General

FRAMED structures are of two main types—pin-pointed and rigid. If the axes of all the individual bars which are joined together to form a frame lie in one plane, the structure is termed a *plane frame*; if they lie in more than one plane, we have a *space frame.* In the present chapter we shall consider only plane frames which are pin-jointed, i.e., the joints are such as to offer no restraint to rotation of the members (see Fig. 8.1). We shall also assume, in general, that external forces are applied only at the joints or nodes of the frame.

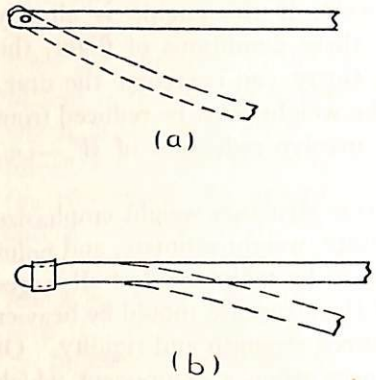


FIG. 8.1.—EFFECT OF END FIXING: (*a*) "PINNED"; (*b*) "FIXED."

If a frame has sufficient members to enable it to carry any system of loads without distortion it is known as a *perfect frame* or a *simply-stiff frame*; since the loads in the members of such a frame may be determined by the methods of statics it is sometimes called, also, a *statically-determinate frame.* A frame which has less than this number of members is a *deficient frame*, and a frame having more than this number of members is called a *redundant frame* or a *statically-indeterminate frame.* Examples of such frames are shown in Fig. 8.2. It should be noted that frames of the type shown in

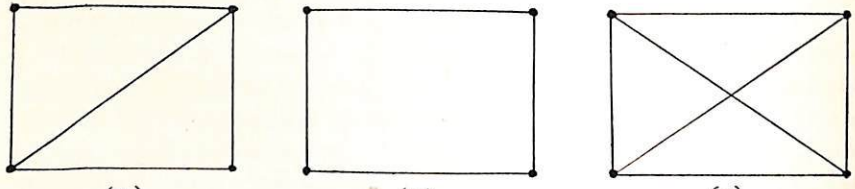


FIG. 8.2.—PLANE FRAMES: (*a*) PERFECT; (*b*) DEFICIENT; (*c*) REDUNDANT.

Fig. 8.2 (c) may occur in aircraft structures with the diagonal members of wire capable of carrying tension only. In such a case, when one wire is under tension, the other will become slack and inoperative, making the frame of the type shown in Fig. 8.2 (a).

8.2. Struts and Ties—Notation

A member of a frame which is in compression is known as a *strut*, and a member in tension is a *tie*. It is usual to denote struts and ties in a frame by inserting arrows indicating the direction of the *loads applied by the members to the joints*. For example, if AB in Fig. 8.3 (a) represents a strut between joints A and B, the strut will be resisting the forces tending to compress it and will,

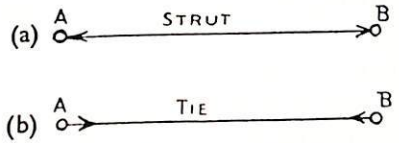


FIG. 8.3.—NOTATION.

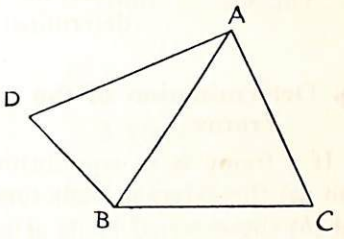


FIG. 8.4.—BUILDING UP A PERFECT PLANE FRAME.

consequently, exert forces on A and B tending to push the joints apart as shown. A tie exerts forces tending to pull the joints together, and the arrows would be inserted as in Fig. 8.3 (b).

No difficulty will be experienced with the notation if it is always remembered that the arrow indicates the direction of the load applied *to the joint*, *not* to the member.

8.3. Relationship between Number of Joints and Number of Members

It will be apparent that the simplest perfect plane frame is a triangle, such as ABC in Fig. 8.4, in which there are three joints and three members. If we wish to brace another joint, D, to the basic triangle, we require two further members, as shown in the figure, and any additional joint will require two more members to brace it.

Hence, if there are j joints altogether, we require three members to form our basic triangle with three of these joints, and two members

for each joint after the first three, so that the number of members, n, is given by

$$n = 3 + 2(j - 3)$$

i.e.,

$$n = 2j - 3$$

In general, this rule will not apply if the structure is not triangulated. For example, although the structure shown in Fig. 8.5 has the correct number of members for a simply-stiff frame, it is not statically determinate. The upper portion would, in fact, collapse under load and is deficient, whilst the lower portion is redundant. Transference of one of the diagonal members would make this frame statically determinate.

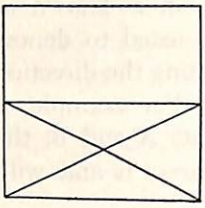


FIG. 8.5.

8.4. Determination of the Loads in the Members of a Plane Frame

If a frame is in equilibrium under a system of external loads then (*a*) the external loads form a system of forces in equilibrium, and (*b*) the external loads at any joint of the frame, together with the internal loads in the members meeting at that joint, form a system of forces in equilibrium.

Application of these two conditions (*a*) and (*b*) enables the loads in the members of a statically determinate frame to be found. Three methods of applying these conditions are in general use and will be discussed in the following paragraphs.

8.5. Method of Resolution

This method consists of writing down the equations expressing (*b*) above for each joint and solving them. In a simple case, direct resolution of the forces may be possible, and it may sometimes be convenient to draw the triangle and polygon of forces for each joint (see Appendix). This method will be illustrated by the following example :—

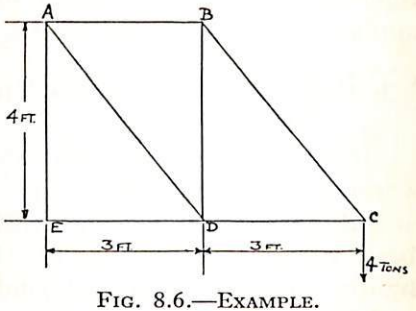


FIG. 8.6.—EXAMPLE.

Example. The frame ABCDE is pinned to the wall at A and E, and carries a load of 4 tons at C as shown. All the joints are pin-joints (Fig. 8.6).

Denote the loads in the members by P_{AB}, P_{BC}, etc., and assume, initially, that all members are in tension.

$$\cos \hat{DCB} = \frac{3}{5}: \quad \sin \hat{DCB} = \frac{4}{5}: \quad \tan \hat{DCB} = \frac{4}{3}$$

At joint C, resolving vertically,

$$P_{BC} \sin \hat{DCB} - 4 = 0$$

i.e.,
$$\frac{4}{5} \times P_{BC} = 4$$

$$P_{BC} = 5 \text{ tons}$$

resolving horizontally,

$$P_{BC} \cos \hat{DCB} + P_{CD} = 0$$

$$P_{CD} = - P_{BC} \times \frac{3}{5} = - 5 \times \frac{3}{5} = - 3 \text{ tons}$$

The negative sign for P_{CD} indicates that our assumption that P_{CD} was tensile was incorrect. CD is, actually, in compression, and we must remember that at D the load in CD acts towards the left.

Consider now joint B, we have, resolving vertically,

$$P_{BC} \sin \hat{DCB} + P_{BD} = 0$$

i.e.,
$$P_{BD} = - 5 \times \frac{4}{5} = - 4 \text{ tons}$$

resolving horizontally,

$$P_{AB} - P_{BC} \cos \hat{BCD} = 0$$

$$P_{AB} = 5 \times \frac{3}{5} = 3 \text{ tons}$$

At joint D, resolving vertically,

$$P_{AD} \sin \hat{ADE} - P_{BD} = 0$$

$$P_{AD} = 4 \times \frac{5}{4} = 5 \text{ tons}$$

(Note that we have used $P_{BD} = + 4$, applying this force in its correct direction as given by compression in BD.)

resolving horizontally,

$$P_{DE} + P_{CD} + P_{AD} \cos A\hat{D}E = 0$$

$$P_{DE} = -3 - 5 \times \frac{3}{5} = -6 \text{ tons}$$

Hence the loads in all the members are

AB—3 tons tension
BC—5 ,, tension
CD—3 ,, compression
DE—6 ,, compression
AD—5 ,, tension
BD—4 ,, compression.

8.6. The Stress Diagram—Bow's Notation

We know that if a number of forces meeting at a point can be represented in magnitude and direction by the sides of a polygon, taking the forces in order round the point, then the system is in equilibrium. Conversely, a system of concurrent forces in equilibrium can be represented in magnitude and direction by the sides of a polygon taken in order round the point of concurrency. If there are only three forces, of course, this becomes the well-known triangle of forces.

The *stress diagram* is merely an application of the polygon of forces, and *Bow's notation*, a convenient method of representing the forces, both external and internal, is as follows :—The spaces between the external forces and between the members of the frame are lettered as shown in Fig. 8.7 (a). Any external force or the force in any member can then be denoted by the two letters on either side of it. For example, the force of 100 lb. applied at joint No. 1 can be represented as AB, and the force applied by member 12 to the same joint may be denoted by BG. Attention is drawn to the fact that, in both cases, the order in which the letters are used is the same—in this case going in a clockwise direction round the joint. The letters used for both forces could have been reversed, but we could not refer to the forces as AB and GB. It is essential to remember that we must continue to go round every joint in the same direction as we commence with the first force.

To avoid confusion it may be as well to take the letters clockwise round every joint, and we shall adhere to this convention in this book. Hence, the force BG represents the force applied by the

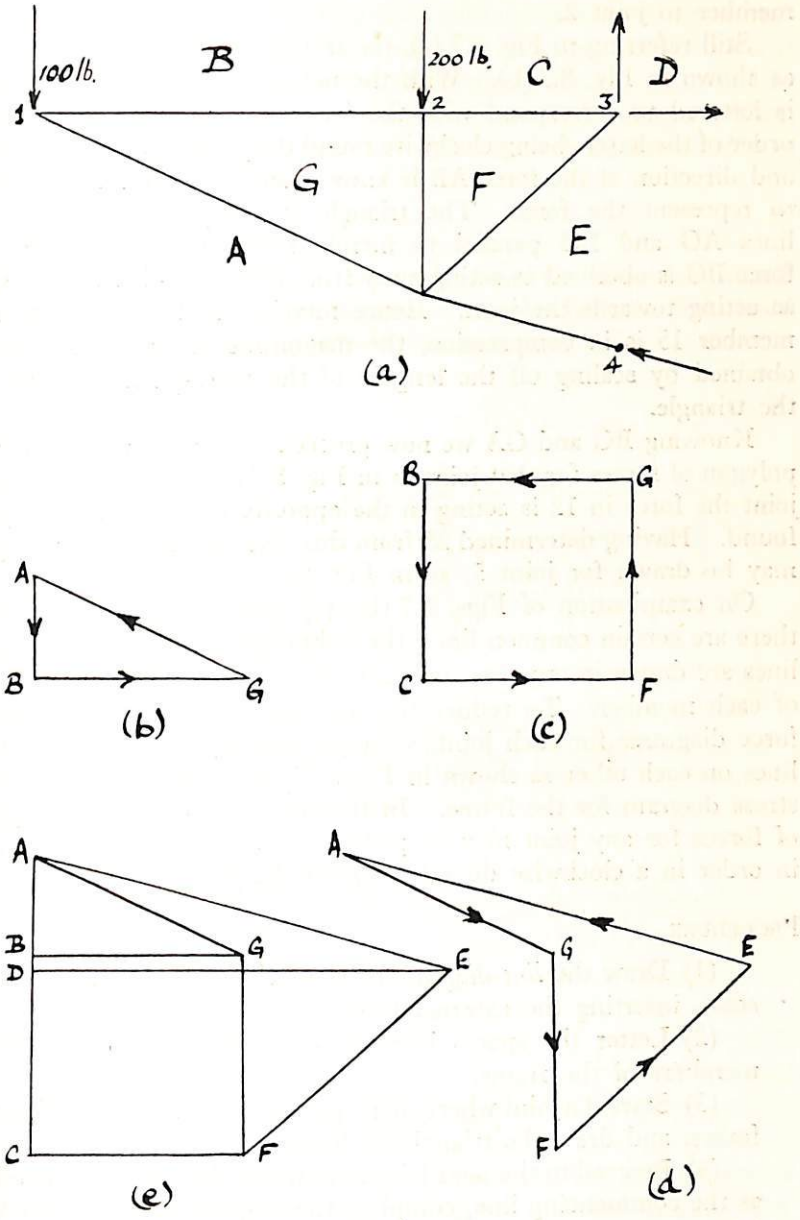


FIG. 8.7.—STRESS DIAGRAM.

member 12 to joint 1, and GB is the force applied by the same member to joint 2.

Still referring to Fig. 8.7 (a), the triangle of forces for joint 1 is as shown in Fig. 8.7 (b). With the notation adopted, the triangle is lettered to correspond with the forces as explained above, the order of the letters being clockwise round the joint. The magnitude and direction of the force AB is known, and the line AB is drawn to represent the force. The triangle is completed by drawing lines AG and BG parallel to members 15 and 12. Then the force BG is obtained as acting away from joint 1, and the force GA as acting towards the joint. Hence member 12 is in tension and member 15 is in compression, the magnitude of the forces being obtained by scaling off the lengths of the corresponding sides of the triangle.

Knowing BG and GA we now proceed to joint 2, and draw the polygon of forces for that joint as in Fig. 8.7 (c), noting that at this joint the force in 12 is acting in the opposite direction to that just found. Having determined 25 from this diagram, a similar polygon may be drawn for joint 5, as in Fig. 8.7 (d).

On examination of Figs. 8.7 (b), (c) and (d), we observe that there are certain common lines, the only difference being that these lines are drawn in opposite directions for the two joints at the ends of each member. To reduce the time spent in drawing separate force diagrams for each joint, we may superimpose corresponding lines on each other as shown in Fig. 8.7 (e), obtaining a composite stress diagram for the frame. In this stress diagram, the polygon of forces for any joint may be picked out by following the letters in order in a clockwise direction round the joint.

PROCEDURE

(1) Draw the *line-diagram* or *space diagram* of the frame, to scale, inserting the external loads.

(2) Letter the spaces between external loads and between members of the frame.

(3) Start at a joint where there are not more than *two* unknown forces, and draw the triangle of forces for that joint.

(4) Proceed to the next joint and, using the common member as the commencing line, complete the polygon of forces for this joint.

(5) Repeat (4) for other joints.

Note.—If at any joint there are more than *two* unknown forces, the polygon of forces cannot be drawn until the number of unknowns has been reduced to not more than two. In some cases this reduction may be performed by use of the *Method of Sections*, described in para. 8.7. Other means of proceeding with a stress diagram when a joint is reached at which there are more than two unknown forces will be given later in the course.

8.7. Method of Sections

This method is very useful when the loads are to be determined in only a small number of members of a complex frame. It is based on the fact that any portion of a frame is in equilibrium under the

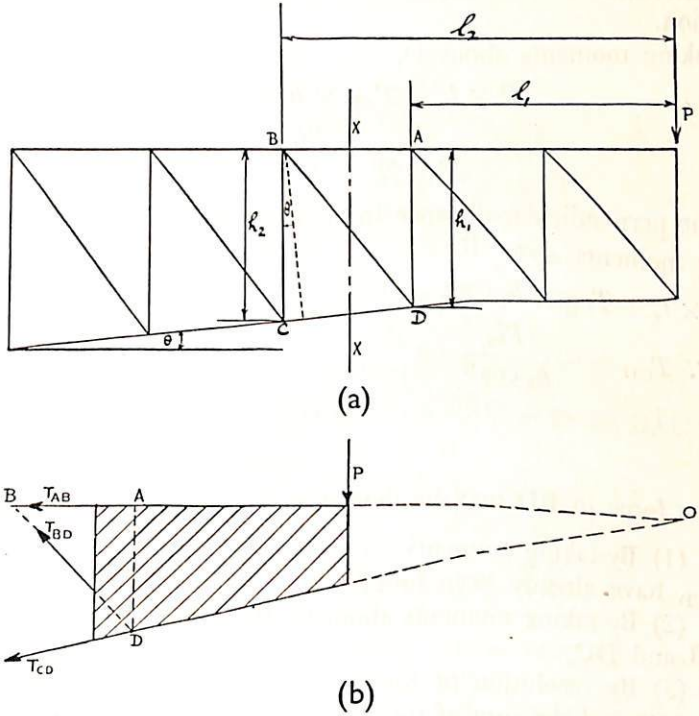


FIG. 8.8.—METHOD OF SECTIONS.

action of the external forces on that portion and the internal forces in the members joining it to the remainder of the frame. For example, if the members AB, CD, BD of the frame in Fig. 8.8 (a) are imagined cut by the section XX as shown, the portion to the

right of XX may be regarded as a rigid body under the action of the force P and the forces in members AB, CD, BD, as indicated in Fig. 8.8 (b).

Since, initially, it is not known whether the loads in the " cut " members are tensile or compressive, it is convenient to assume that these loads are all tensile, the directions of the forces applied to the separated portion of the frame then being as shown in the figure.

The forces T_{AB}, T_{BD}, T_{CD}, may now be determined by applying the conditions of equilibrium of forces. The sum of the moments of these forces and P about any point must be zero.

By taking moments about the intersection of any two of the unknown forces, these two will be eliminated from the moment equation since a force has no moment about a point on its line of action.

Taking moments about D,

$$P \times l_1 - T_{AB} \times h_1 = 0$$

$$\therefore\ T_{AB} = \frac{Pl_1}{h_1}$$

The perpendicular distance from B to CD is BC $\cos\theta$, so that, taking moments about B,

$$P \times l_2 + T_{CD} \times h_2 \cos\theta = 0$$

$$\therefore\ T_{CD} = -\frac{Pl_2}{h_2 \cos\theta}$$

(The negative sign indicates that the force in CD is compressive, not tensile as assumed initially.)

The force in BD may be determined by several methods :—

(1) By taking moments about A or C, provided that T_{AB} and T_{CD} have already been found.

(2) By taking moments about O, the point of intersection of AB and DC.

(3) By resolution of forces—the sum of the vertical components and the sum of the horizontal components of the four forces T_{AB}, T_{BC}, T_{CD} and P must be zero.

It may be noted that the loads in the vertical members AD and BC may be found by using sections such as YY and ZZ. A numerical example illustrating this method follows.

Example. Fig. 8.9 shows part of the side frame of a truss type fuselage. Using the method of sections, determine the loads in the members AB, BC, CD, DA and AC, when there is a vertical load of 600 lb. applied as shown.

It is desirable, first, to obtain various dimensions which will be required. The depth of the fuselage side decreases by 3 ft. over a length of 12 ft.—i.e., the depth decreases at the rate of 1 ft. in a length of 4 ft. Hence AD is 6 in. shorter than EF and BD is 1 ft. less than EF.

$$\therefore \text{AD} = 3{\cdot}5 \text{ ft.}$$

and

$$\text{BC} = 3 \text{ ft.}$$

Similarly, we observe that the distance GO = 4 ft.
From the triangle CHE,

$$\tan\theta = \frac{\text{CH}}{\text{HE}} = \frac{1}{4}$$

$$\cos\theta = \frac{\text{HE}}{\text{CE}} = \frac{4}{\sqrt{4^2 + 1^2}} = \frac{4}{\sqrt{17}}$$

AJ is drawn perpendicular to DC

$$\text{AJ} = \text{AD}\cos\theta = 3{\cdot}5 \times \frac{4}{\sqrt{17}} = 3{\cdot}39 \text{ ft.}$$

OK is drawn perpendicular to AC.

$$\text{OK} = \text{OA}\sin\text{B}\hat{\text{A}}\text{C} = 14\sin\text{B}\hat{\text{A}}\text{C}$$

But

$$\tan\text{B}\hat{\text{A}}\text{C} = \frac{\text{BC}}{\text{BA}} = \frac{3}{2}$$

$$\therefore \sin\text{B}\hat{\text{A}}\text{C} = \frac{3}{\sqrt{3^2 + 2^2}} = \frac{3}{\sqrt{13}}$$

$$\therefore \text{OK} = 14 \times \frac{3}{\sqrt{13}} = 11{\cdot}65 \text{ ft.}$$

We now have all the dimensions necessary to determine the forces required.

Consider the portion of the frame to the right of section XX

Assume T_{AB}, T_{AC}, T_{CD} are all tensile.

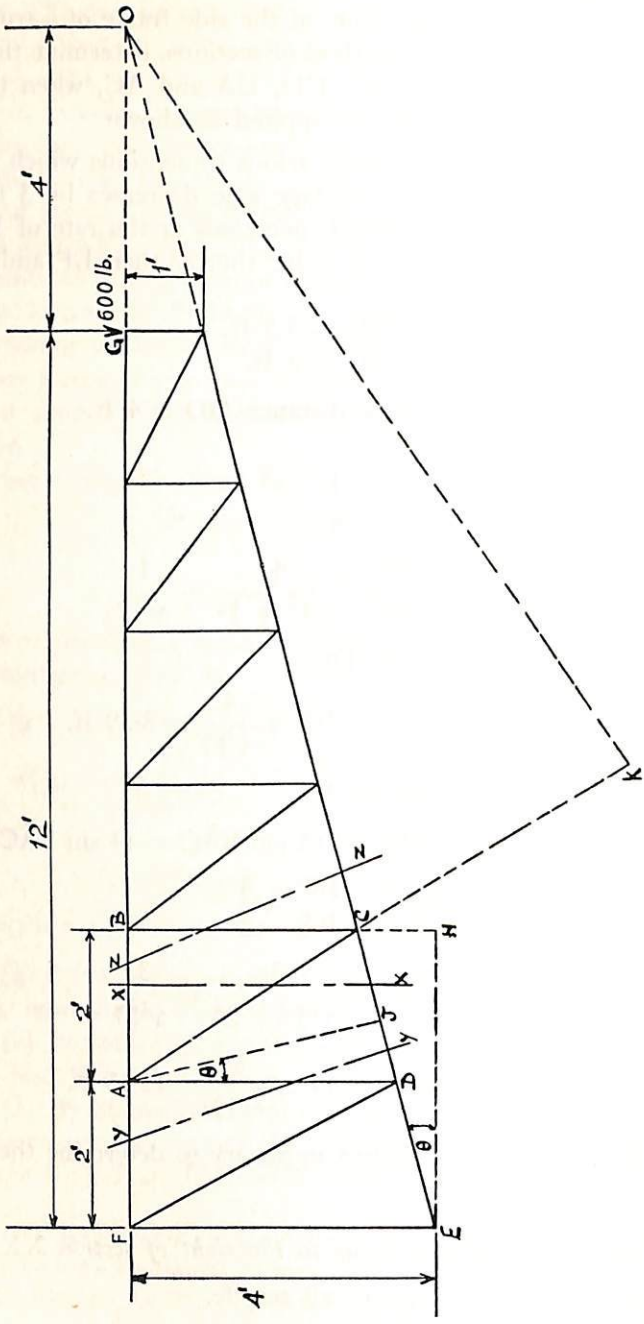


FIG. 8.9.—EXAMPLE ILLUSTRATING METHOD OF SECTIONS.

Taking moments about C,

$$600 \times 8 - T_{AB} \times 3 = 0$$

$$T_{AB} = \frac{600 \times 8}{3} = \mathbf{1600\ lb.}$$

Taking moments about A,

$$600 \times 10 + T_{CD} \times 3{\cdot}39 = 0$$

$$T_{CD} = -\frac{600 \times 10}{3{\cdot}39} = \mathbf{-1770\ lb.}$$

Taking moments about O

$$600 \times 4 - T_{AC} \times 11{\cdot}65 = 0$$

$$T_{AC} = \frac{600 \times 4}{11{\cdot}65} = \mathbf{206\ lb.}$$

Consider the portion of the frame to the right of section YY

Taking moments about O,

$$600 \times 4 + T_{AD} \times 14 = 0$$

$$T_{AD} = -\frac{600 \times 4}{14} = \mathbf{-171{\cdot}5\ lb.}$$

Consider the portion of the frame to the right of section ZZ

Taking moments about O,

$$600 \times 4 + T_{BC} \times 12 = 0$$

$$T_{BC} = -\frac{600 \times 4}{12} = \mathbf{-200\ lb.}$$

Summarising the results, we have :—

Member.	Load – lb.
AB	1600 Tension
BC	200 Compression
CD	1770 Compression
DA	171·5 Compression
AC	206 Tension.

Examples, Chapter 8

1. Calculate the loads in the members of the pin-jointed frame shown in Fig. 8.10.

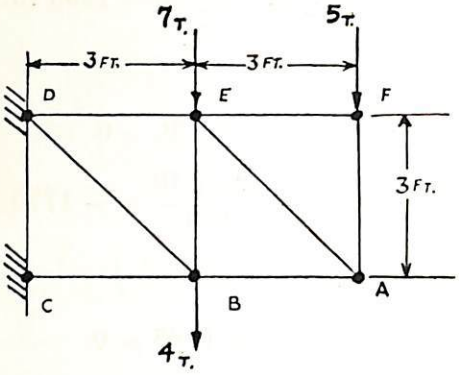


FIG. 8.10.

2. Draw a stress diagram to determine the loads in the members of the engine mounting in Fig. 8.11.

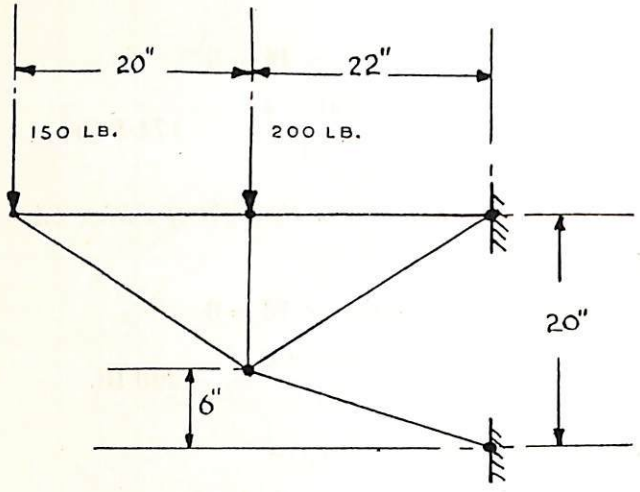


FIG. 8.11.

3. Obtain the loads in the biplane lift truss shown in Fig. 8.12. Note that the wire bracing can withstand tension only, so that the wires which would be in compression may be neglected.

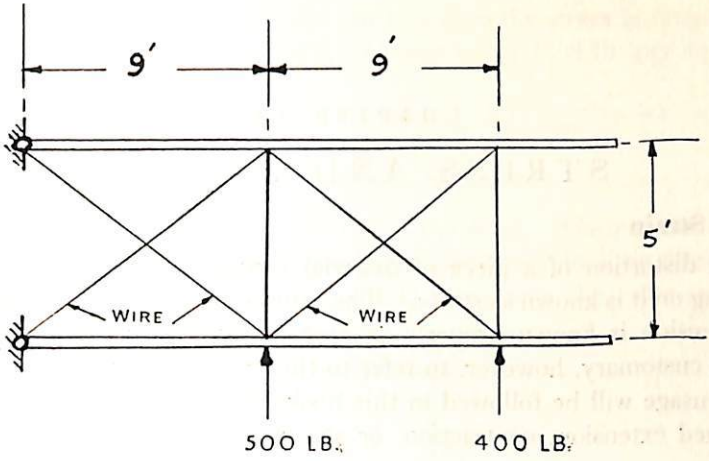


FIG. 8.12.

4. Fig. 8.13 shows one side of a braced fuselage. Use the method of sections to find the loads in members AB, BC, CD, DA and AC.

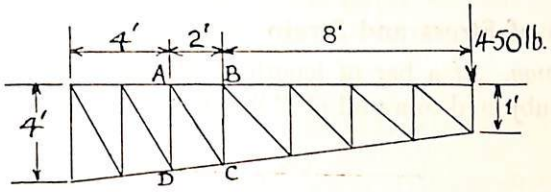


FIG. 8.13.

5. Determine the loads in members AB, AC, CD of the framework in Fig. 8.14.

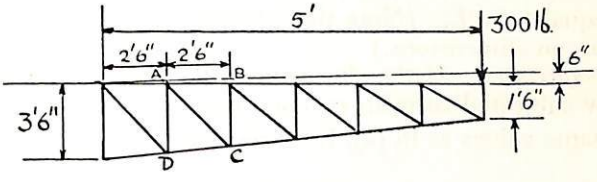


FIG. 8.14.

CHAPTER IX

STRESS AND STRAIN

9.1. Strain

THE distortion of a piece of material consequent upon the forces acting on it is known as *strain*. The ratio of the strain to the original dimension is known properly as *proportional strain* or *unit strain*. It is customary, however, to refer to this ratio simply as strain, and this usage will be followed in this book, the actual distortion being termed extension, contraction, or shear, as the case may be.

9.2. Stress

The internal forces in a piece of material under strain constitute *stress*. Throughout the text the term "stress" will be used to mean stress per unit area (intensity of stress, or proportional stress, or unit stress).

9.3. Types of Stress and Strain

(a) *Tension*. If a bar of length L in. and cross-sectional area A sq. in., subjected to a pull of P lb. (Fig. 9.1), extends an amount

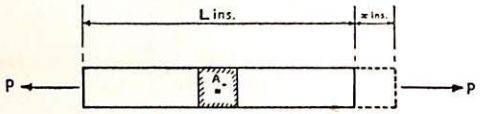


FIG. 9.1.—SIMPLE TENSION.

x in., the *tensile stress* is equal to P/A lb. per sq. in., and the *tensile strain* is equal to x/L. (Note that strain, being length divided by length, has no dimensions.)

(b) *Compression*. If the direction of P be reversed so that the bar is now subjected to push or compression, the stress and strain have the same values as in (a), but they are now *compressive* instead of tensile.

(c) *Shear* arises when the externally applied forces are such that portions of the material tend to slide over one another; for example, in Fig. 9.2 (a), the upper and lower proportions of the rivet tend to

slide as shown in Fig. 9.2 (b). In this case the stress is tangential to the surface on which it acts, its value being P/A lb. per sq. in.,

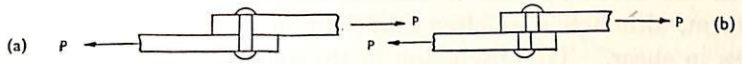


FIG. 9.2.—SHEARING OF RIVET.

where A is the cross-sectional area of the rivet. *Shear strain* is an angular strain, the concept of which may be understood by reference to Fig. 9.3, where the square ABCD represents the cross-section

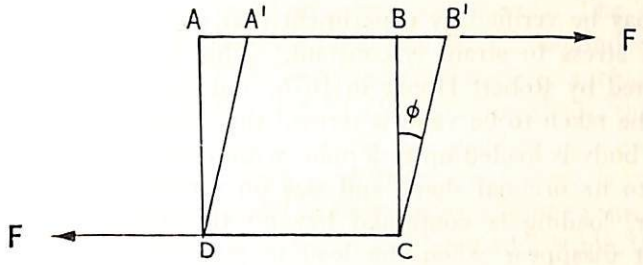


FIG. 9.3.—DEFINITION OF SHEAR STRAIN.

of a block which is subjected to the forces F. Assuming CD to remain fixed, the section will take the form shown by A'B'CD. The shear strain is then :—

$$\frac{BB'}{BC} = \tan \phi = \phi, \text{ when } \phi \text{ is small.}$$

(d) *Bearing.* In Fig. 9.4, the load P is transmitted from one plate to the other by the rivet R. The intensity of pressure between

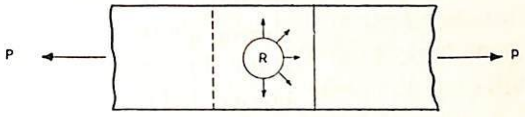


FIG. 9.4.—BEARING.

the rivet and the plate is known as the *bearing stress*, the maximum value being P/dt, where d is the rivet diameter and t the thickness of the plate.

9.4. Uniformity of Stress Distribution

In the case of direct stresses (tensile and compressive) due to axial loads, it is usual to assume that the stress is distributed uniformly

over the area on which it acts, so that the stress everywhere is P/A.

In the case of shear or bearing stress the distribution is not uniform, although it is often sufficient to consider only the mean stress in shear. The discussion of the actual distribution of stress will be left until a later stage.

We shall see in a future chapter that bending gives rise to tensile and compressive stresses which are not uniformly distributed.

9.5. Hooke's Law

It may be verified by experiment that within certain limits the ratio of stress to strain is constant. This experimental law was first stated by Robert Hooke in 1676, and the range within which it may be taken to be valid is termed the *elastic range.*

If a body is loaded up to a point within the elastic range, it will return to its original shape and size on removal of the load. If, however, loading is continued beyond this range, all the strain will not disappear when the load is removed, and a *permanent set* will result.

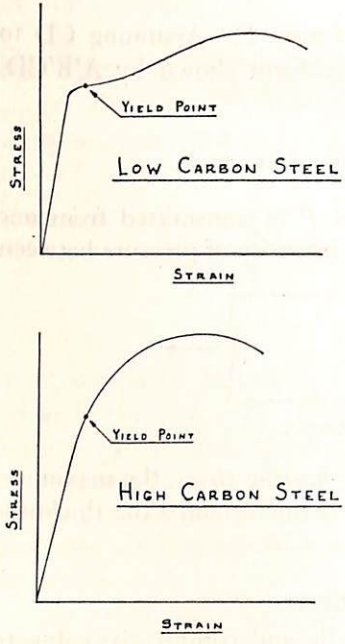


Fig. 9.5.—Typical Stress–Strain Diagrams.

9.6. Tensile Properties

When a specimen of material is tested in tension, it is usual to plot the results of the test in the form of a *stress–strain diagram.* An understanding of this diagram is essential since it forms the basis of most strength requirements. Fig. 9.5 shows some typical diagrams, and some important properties are discussed below.

Young's Modulus (E). It will be noted that the first part of the diagram is a straight line, verifying Hooke's law. The numerical value of the constant ratio of stress to strain over this portion of the diagram is known as *Young's modulus* or the *modulus*

of elasticity in tension, and it is denoted by E. Since strain is non-dimensional, E has the same dimensions as stress.

Tensile Elastic Limit or Proportional Limit. This is the stress at which the diagram begins to depart from a straight line. Reference to the figure shows that it is practically impossible to determine this stress accurately, and it is usual to quote the stress at which a small permanent strain is obtained. The amount of permanent strain usually allowed for is 0·0001.

Tensile yield stress. The diagram for low-carbon steel shows that at a point above the elastic limit the specimen elongates considerably with little or no increase in stress. The point at which this occurs is the *yield point* and the corresponding stress is the *yield stress*. With many other materials, such as non-ferrous materials, high-carbon steels and alloy steels, this sharp break in the curve does not appear, the yielding of the material taking place more gradually. In these cases there is no easily determined yield point, and in order to fix a limit to the amount of permanent strain that may be considered admissible, the yield stress is usually quoted as that stress corresponding to a permanent strain of 0·002. This stress is often referred to as the *0·2% proof-stress*, being the stress corresponding to a strain of 0·2%. It may be determined from the stress–strain diagram as the intersection of the curve with a line drawn parallel to the straight line portion of the diagram through the point representing zero stress and 0·002 strain.

Ultimate Tensile Stress. This is the stress at the maximum load reached in the test, and it is based, as are all the other stresses plotted in the diagram, on the original cross-sectional area of the specimen. Because of this means of evaluating stress, the specimen appears to fail at a stress lower than the maximum, but as a lateral contraction of the specimen actually takes place, the true stress at failure is the greatest true stress.

9.7. Compressive Properties

The results of compression tests on short compact specimens may be plotted as stress–strain diagrams similar to those for tension. For most materials the modulus of elasticity in compression is approximately equal to that in tension.

The elastic limit and yield stress may be determined as in tensile tests, but the ultimate compressive stress presents more difficulty.

A definite value of the ultimate stress may be obtained for short compact specimens of stone, wood and some metals, which will fail by fracture in compression, but most metals simply yield and swell out so that the increasing load is supported by an increasing area, and it is necessary to adopt some arbitrary criterion, the assumption being commonly made that the ultimate compressive stress is equal to the ultimate tensile stress.

However, we shall see later, in aircraft structures, that compression members will usually fail by " buckling " at a stress much lower than the ultimate.

9.8. Shear Properties

The *modulus of elasticity in shear*, often called the *modulus of rigidity*, and denoted in this book by G, may be determined from a stress–strain diagram, obtained by twisting a round tube and plotting the torsional shear stress (see Chap. XII), against the angular strain, as the ratio of shear stress to shear strain.

Other properties obtained from such a diagram cannot be regarded as properties of the material since they depend on the shape of the specimen.

9.9. Poisson's Ratio

Axial strain of a piece of material is always accompanied by lateral strain of opposite sign. For example, if a bar is extended under tension, it becomes thinner ; in this case a tensile axial strain is accompanied by a compressive lateral strain. Within the elastic range the ratio of lateral strain to longitudinal (axial) strain is a constant, known as *Poisson's ratio*, denoted in this book by σ. For steels and aluminium alloys the value of σ is usually between $\frac{1}{4}$ and $\frac{1}{3}$.

9.10. Relation between the Elastic Constants

The constants E, G, σ, are often termed the *elastic constants* of a material, and certain relationships may be shown to exist between them. These relations, the derivation of which will be postponed to a later stage, are as follows :—

$$G = \frac{E}{2(1 + \sigma)}$$

With $\sigma = \frac{1}{4}$, this gives $G = \frac{2}{5}E$, and this may be taken as the approximate value of G if accurate data are not available.

9.11. Examples on Calculation of Stress and Strain

Example 1. A bar of diameter $\frac{1}{2}$ in. carries a tensile load of 6500 lb. What is the tensile stress? If the initial length of the bar is 6 in. and the extension under this load is 0·02 in., calculate the tensile strain.

$$\text{Cross-sectional area of bar} = \frac{\pi}{4} \times (\tfrac{1}{2})^2 = 0{\cdot}196 \text{ sq. in.}$$

$$\text{Tensile stress} = \frac{\text{Load}}{\text{Area}} = \frac{6500}{0{\cdot}196} = \mathbf{33{,}200 \text{ lb. per sq. in.}}$$

$$\text{Tensile strain} = \frac{\text{Extension}}{\text{Original length}} = \frac{0{\cdot}02}{6} = \mathbf{0{\cdot}00333}$$

Example 2. In a tensile test a specimen of 0·5 sq. in. cross-sectional area and 8 in. in length extended an amount 0·01 in. under a load of 7·5 tons. Calculate the tensile stress and strain and the value of Young's modulus for the material.

$$\text{Stress} = \frac{7{\cdot}5}{0{\cdot}5} = \mathbf{15 \text{ tons per sq. in.}}$$

$$\text{Strain} = \frac{0{\cdot}01}{8} = \mathbf{0{\cdot}00125}$$

$$\text{Young's modulus} = E = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{15}{0{\cdot}00125}$$

$$= \mathbf{12{,}000 \text{ tons per sq. in.}}$$

Example 3. Calculate the shear stress on the pin and the bearing stress between the pin and the part A in Fig. 9.6.

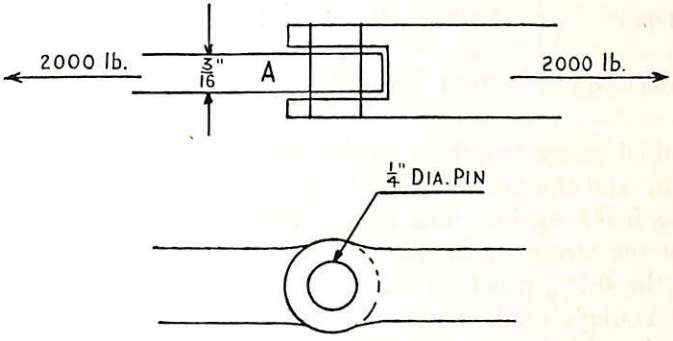


FIG. 9.6.—EXAMPLE.

In this case the pin is said to be in "double shear," the load which is applied by the single lug A being taken equally by the two arms of the fork end fitting, so that there is a shear force of 1000 lb. on the pin.

$$\text{Cross-sectional area of pin} = \frac{\pi}{4} \times (\tfrac{1}{4})^2 = 0{\cdot}049 \text{ sq. in.}$$

$$\text{Shear stress in pin} = \frac{1000}{0{\cdot}049} = \mathbf{20{,}400 \text{ lb. per sq. in.}}$$

$$\text{Thickness of part A} = 0{\cdot}1875 \text{ in.}$$

$$\text{Bearing stress} = \frac{\text{Load}}{\text{Dia.} \times \text{Thickness}} = \frac{2000}{\frac{1}{4} \times 0{\cdot}1875}$$

$$= \mathbf{42{,}650 \text{ lb. per sq. in.}}$$

Examples, Chapter X

1. A bar of length 40 in. and cross-sectional area 0·125 sq. in. is subjected to a pull of 4 tons. Calculate the tensile stress and, if E is 13,000 tons per sq. in., determine the extension.

2. During a test a bar of length 8 in., and cross-sectional area 0·486 sq. in. stretched 0·00775 in. under a pull of 14,110 lb. Find the value of E for the material.

3. In a tensile test on a steel bar the following results were obtained :—

Load (lb.)	1982	3965	7930	11,894	14,650
Extension (in.)	0·00120	0·00242	0·00480	0·00736	0·0090
Load (lb.)	15,859	16,262	16,643	16,710	17,046
Extension (in.)	0·00984	0·01024	0·0108	0·1296	0·1368

The initial gauge length on which the extensions were measured was 8 in. and the cross-sectional area at the commencement of the test was 0·442 sq. in. The load at failure was 25,155 lb.

Plot the stress–strain diagram and determine

(a) the 0·1% proof stress,

(b) Young's modulus,

(c) the ultimate tensile stress.

4. A piece of material is subjected to a shear stress of 10,000 lb. per sq. in. Determine the shear strain if E is 10^7 lb. per sq. in. and Poisson's ratio is 0·25.

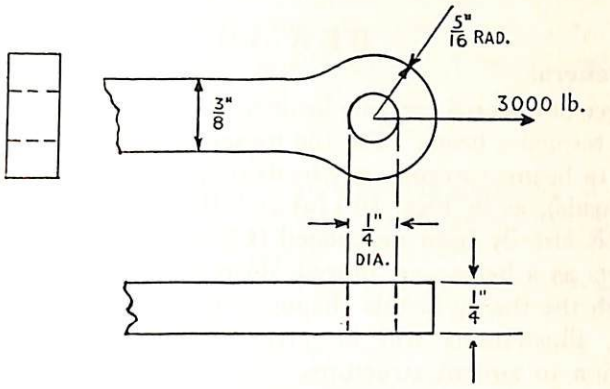


FIG. 9.7.

5. A tensile load of 3000 lb. is transmitted to the fitting shown in Fig. 9.7 by means of a pin 0·25 in. diameter, which is in shear. Calculate the shear stress in the pin, the bearing stress of the pin in the fitting, and the maximum tensile stress in the fitting.

CHAPTER X

BEAMS

10.1. General

ANY piece of material carrying loads which are not purely axial loads may be termed a beam. For the present we shall confine our discussion to beams carrying only loads perpendicular to the axis (i.e., lateral loads), as in Figs. 10.1 (a) and 10.1 (b).

It has already been mentioned (Chap. VII), that an aeroplane wing acts as a beam—as, indeed, do many parts of the structure. Although the theory in this chapter will be presented in a general manner, illustrations will be given, wherever possible, of the application to aircraft structures.

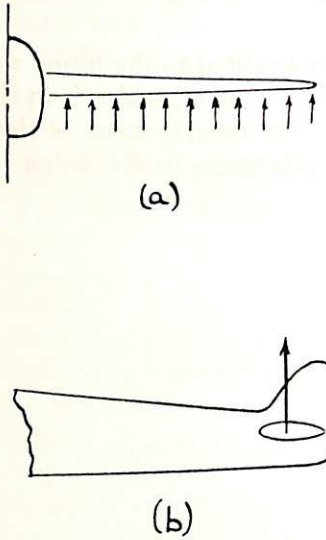


FIG. 10.1.—BEAMS.

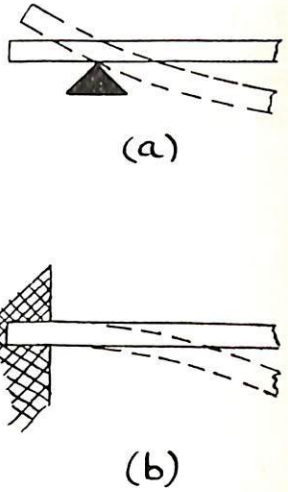


FIG. 10.2.—TYPES OF SUPPORT.

10.2. Types of Support

(a) A beam is said to be *simply supported* when there is no restraint which prevents rotation at the supports (see Fig. 10.2 (a)).

(b) *Built-in* or *encastré* beams are restrained at the supports so that rotation at the supports is wholly or partially prevented (see Fig. 10.2 (b)).

It will be apparent that the difference between the two types of support is that the latter must apply a couple or twisting moment in addition to a force, whilst the former applies a force only.

With the exception of the cantilever, which is a beam built-in at one end (the root end), and completely free at the other end (the tip), we shall consider only simply-supported beams until later in the course.

10.3. Equilibrium of Beams

Suppose a beam of length L to be simply supported at A and B, and to carry loads W_1, W_2, . . . W_n as shown in Fig. 10.3. Let

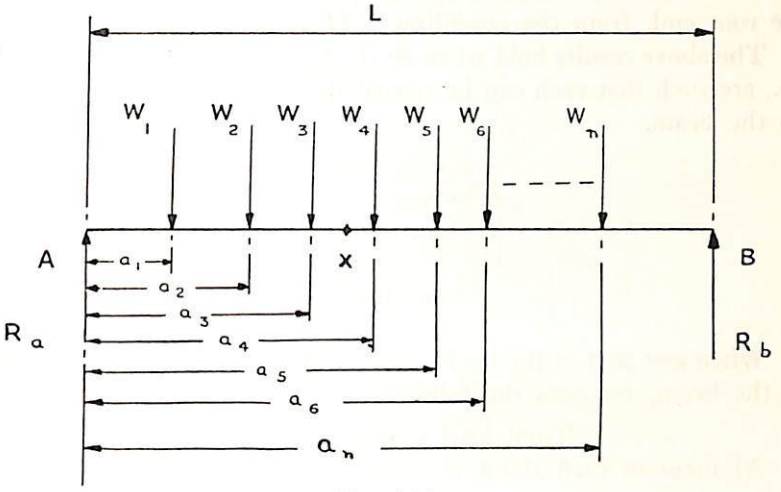


FIG. 10.3.

the reactions at A and B be denoted by R_A and R_B, respectively. Since the system of loads formed by W_1, W_2, . . . W_n, R_A and R_B is in equilibrium, we must have, in the notation of the Appendix, para. A.1.9,

$$\Sigma H = 0\,; \qquad \Sigma V = 0\,; \qquad \Sigma M = 0.$$

The first of these conditions is satisfied automatically, since there are no horizontal forces.

The second leads to the equation :—

$$R_A + R_B - W_1 - W_2 - \ldots - W_n = 0 \quad . \qquad \text{Eq. 10.1}$$

To apply the third condition of equilibrium, take moments about A,

$$W_1a_1 + W_2a_2 + \ldots + W_na_n - R_BL = 0. \qquad \text{Eq. 10.2}$$

From 10.2,

$$R_B = \frac{W_1a_1 + W_2a_2 + \ldots + W_na_n}{L}$$

Substituting in 10.1 then gives the value of R_A. Alternatively R_A may be found by taking moments about B.

If the loads are not all applied between the supports the procedure will be exactly the same, but care must be taken with signs in the moment equation.

In the case of a cantilever, the total reaction at the support must be made up of a force and a couple. The force is obtained from the condition $\Sigma V = 0$, and the couple, by taking moments about the root end, from the condition $\Sigma M = 0$.

The above results hold when all the loads are concentrated loads, i.e., are such that each can be regarded as applied at a single point on the beam.

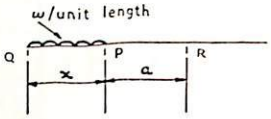


FIG. 10.4.

When any part of the load is spread uniformly over a length PQ of the beam, we note the following:—

Total load $= w \times x = wx$ (see Fig. 10.4)

Moment of load about P $= wx \times x/2$

" " " " R $= wx(x/2 + a)$

i.e., When taking moments of a distributed load we use the fact that the moment of a system of forces about a point is equal to the moment of the resultant force about that point.

If the loads applied to the beam contain both concentrated and uniformly distributed loads, the reactions due to each type of load may be computed separately and added together.

The procedure will be illustrated by the numerical examples which follow.

Examples. Determine the reactions at the supports for each of the beams shown in Fig. 10.5.

(a) *Taking moments about* B,

$R_A \times 6 - 100 \times 5 - 350 \times 2 = 0$

i.e., $R_A \times 6 = 500 + 700 = 1200$

$$R_A = \mathbf{200\ lb.}$$

$$R_A + R_B - 100 - 350 = 0$$

$$R_B = \mathbf{450 - 200 = 250\ lb.}$$

(b) *Taking moments about* B,

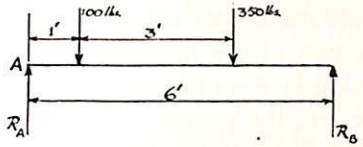
$$R_A \times 5 - 300 \times 2 + 100 \times 2 = 0$$

$$R_A \times 5 = 600 - 200 = 400$$

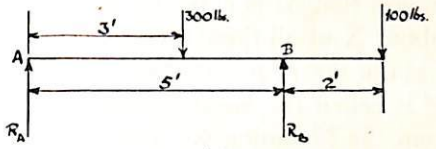
$$R_A = \mathbf{80\ lb.}$$

$$R_A + R_B - 300 - 100 = 0$$

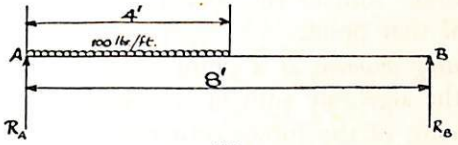
$$R_B = \mathbf{400 - 80 = 320\ lb.}$$



(a)



(b)



(c)

FIG. 10.5.—EXAMPLES.

(c) *Taking moments about* A,

$$R_B \times 8 - (100 \times 4) \times 4/2 = 0$$

$$R_B \times 8 = 800$$

$$R_B = \mathbf{100\ lb.}$$

$$R_A + R_B - 400 = 0$$

$$R_A = \mathbf{400 - 100 = 300\ lb.}$$

10.4. Shear Force and Bending Moment

Consider the portion XB of the beam AB shown in Fig. 10.3. This portion will be in equilibrium under the forces W_4, W_5, W_6, . . . W_n, R_B and the forces exerted on it by the portion XA of the beam. Since any system of forces in one plane can be represented by a single force and a couple, let us suppose that the portion AX exerts on XB a force F and a couple M, as indicated in Fig. 10.6. Applying the conditions of equilibrium to XB shows that F is equal to the sum of all the forces acting between X and B, and that M is equal to the sum of the moments about X of all these forces. Since every action is accompanied by an equal and opposite reaction, the portion XB will exert on AX an equal force F and an equal moment M, and it may be shown in a similar manner that F is equal to the sum of all the forces acting between X and A, and that M is equal to the sum of the moments about X of all these forces. The force F is known as the *shear force* at the point X, and the couple M is called the *bending moment* at the point X. From the foregoing we may define these quantities thus :—

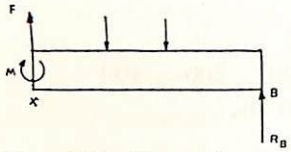


FIG. 10.6.—SHEAR FORCE AND MOMENT BENDING.

The *shear force* at a point on a beam is equal to the algebraic sum of the forces acting on *either side* of that point.

The *bending moment* at a point on a beam is equal to the algebraic sum of the moments about that point of the forces acting on *either side* of the point.

10.5. Sign Conventions

Sign conventions for shear force and bending moments vary, and the following will be adopted in this course :—

Loads will be considered positive when acting upwards.

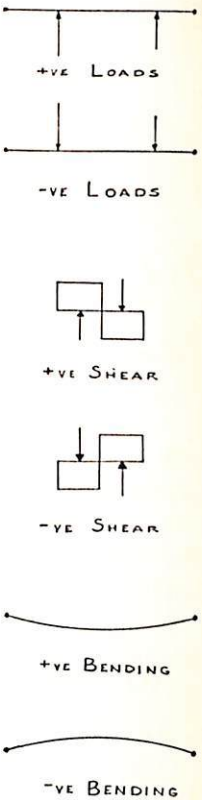


FIG. 10.7.—SIGN CONVENTIONS.

Shear force will be considered positive when the section on the right tends to move downwards.

Bending moment will be considered positive when it causes compression in the upper surface of the beam. In this case the beam bends so that it is concave on the upper surface.

These conventions are illustrated in Fig. 10.7.

10.6. Shear Force and Bending Moment Diagrams

As will be seen later, a knowledge of the variation of shear force and bending moment along a beam is essential to the design of a beam. It is usual to plot these quantities along the span of the beam in the form of shear force and bending moment diagrams. Typical diagrams for certain standard forms of loading are given below as examples.

Case I—Cantilever with Concentrated Load at the Tip

Fig. 10.8 (a) represents a cantilever beam carrying a concentrated load W at the tip, the length of the beam being L.

From para. 10.3, the reaction at the root is made up of the force W and the couple WL, as indicated in the figure.

At any point between the tip and the root the sum of the forces on either side of that point is W, so that the shear force is constant and equal to W over the whole length (see Fig. 10.8 (b)).

At any point distant x from the tip, the bending moment is Wx, so that the bending moment diagram is triangular, as in Fig. 10.8 (c).

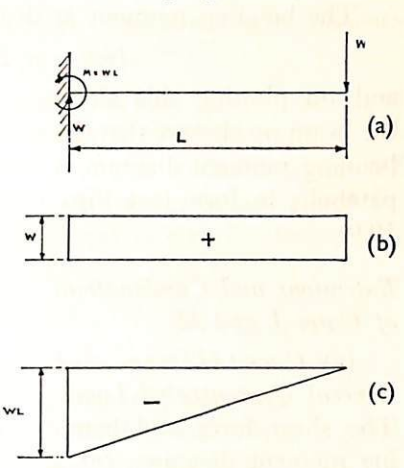


Fig. 10.8.—Cantilever with Concentrated Load.

The signs of shear force and bending moment are as suggested in para. 10.5.

Case II—Cantilever with Uniformly-Distributed Load

Suppose now, the load W, instead of being concentrated at the tip as in Case I, is spread uniformly over the whole length of the beam, giving a loading of w $(= W/L)$ per unit length.

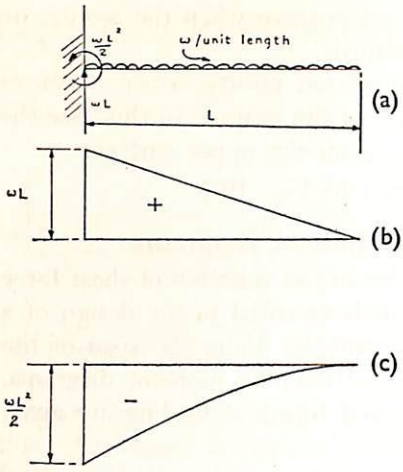


FIG. 10.9.—CANTILEVER WITH UNIFORMLY-DISTRIBUTED LOAD.

The moment of this load about the root is now

$$wL \times L/2 = wL^2/2,$$

so that the couple applied at the root must now be $WL/2$ instead of WL, as in Case I. The force applied by the support is still W $(= wL)$.

At any point distant x from the tip, the total load between that point and the tip is wx, so that the shear force diagram is triangular in form, the shear force itself increasing at a uniform rate from tip to root.

The bending moment at distance x from the tip is

$$(wx \times x/2) = wx^2/2,$$

and on plotting this along the beam we observe that the bending moment diagram is parabolic in form (see Figs. 10.9).

Extensions and Combinations of Cases I and II

(a) *Cantilever with Several Concentrated Loads.* The shear force and bending moment diagrams for a cantilever loaded as shown in Fig. 10.10 (a) are given in Figs. 10.10 (b) and (c). These may be obtained by drawing the diagrams for each load separately and adding to obtain the resultant diagrams, or directly from our definitions of shear force

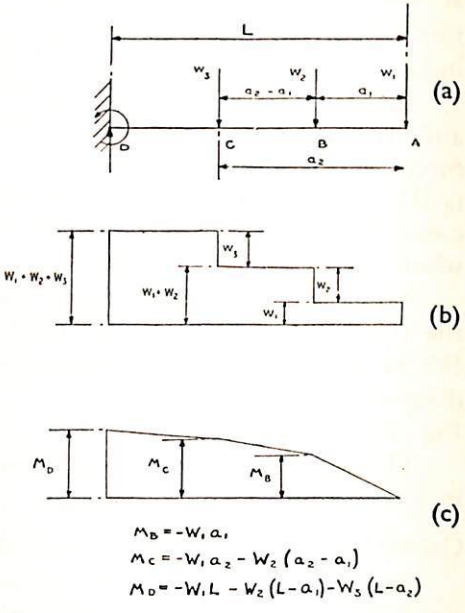


FIG. 10.10.—CANTILEVER WITH SEVERAL CONCENTRATED LOADS.

and bending moment. Thus, between A and B, the shear force is W_1, between B and C it is $W_1 + W_2$, and between C and D it is $W_1 + W_2 + W_3$, this being also the value of the force applied by the support.

Measuring x from A, the bending moment between A and B is W_1x, between B and C it is $W_1x + W_2(x - a_1)$, and between C and D it is $W_1x + W_2(x - a_1) + W_3(x - a_2)$, the maximum value, equal to the couple at the root, being $W_1L + W_2(L - a_1) + W_3(L - a_2)$.

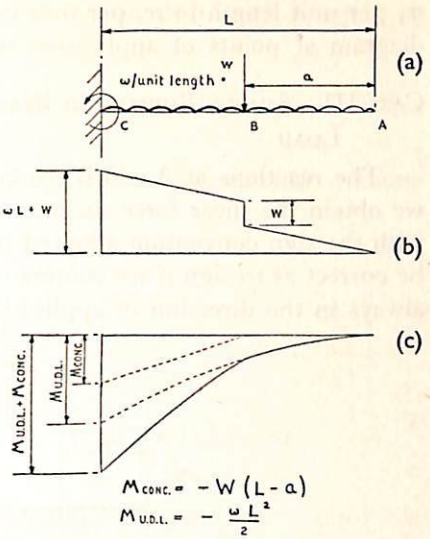


FIG. 10.11.—CANTILEVER WITH COMBINED LOADS.

(b) *Cantilever with Uniformly-Distributed Load and Concentrated Load.* The diagrams shown in Fig. 10.11 are obtained as follows. Between A and B the shear force is wx. At B the shear force is increased by an amount W, equal to the concentrated load applied there. Between B and C the shear force is $wx + W$.

Between A and B the bending moment is $wx^2/2$. Between B and C it is $wx^2/2 + W(x - a)$.

Discontinuities in Shear Force and Bending Moment Diagrams

The student will find it useful to remember the following properties of shear force and bending moment diagrams :—

A " step " is produced in a shear force diagram only at a point where a concentrated load is applied, and the amount by which the diagram is stepped up or down is equal to the magnitude of the load producing the step.

Similarly, a step is produced in a bending moment diagram only at a point where a concentrated couple is applied (e.g., at the root of a cantilever).

FIG. 10.12.—DISCONTINUITIES.

Discontinuities of the types shown in Fig. 10.12, occur in a shear force diagram at points where a distributed load changes from, say,

w_1 per unit length to w_2 per unit length, and in a bending moment diagram at points of application of concentrated loads.

CASE III—SIMPLY-SUPPORTED BEAM WITH CENTRAL CONCENTRATED LOAD

The reactions at A and B are each $W/2$, and commencing at A, we obtain the shear force diagram as in Fig. 10.13 (b). Note that, with the sign convention adopted in para. 10.5, the shear force will be correct as to sign if we commence at the left-hand end and draw always in the direction of application of the various loads—e.g., at

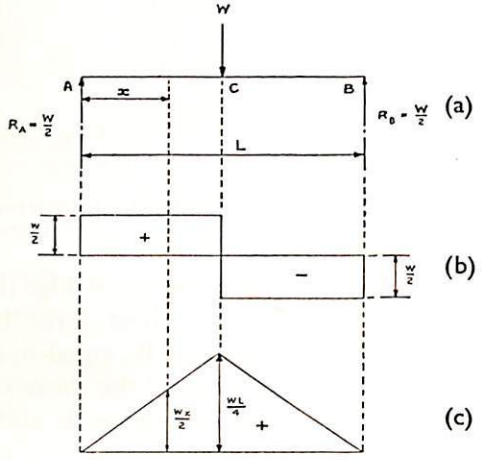


FIG. 10.13.—SIMPLY-SUPPORTED BEAM WITH CENTRAL CONCENTRATED LOAD.

A the load $W/2$ acts upwards, so that we draw the diagram upwards that amount; at C the load W acts downwards so we draw a step downwards an amount W; and at B we again go up an amount $W/2$ to return to zero. Between B and C, the total force on the right is upwards, giving a negative shear force; between C and A, the total force on the right is downwards, giving a positive shear force.

The bending moment diagram is shown in Fig. 10.13 (c). Between A and C, at distance x from A, the bending moment is $\frac{W}{2}x$ (positive). Between C and B, at distance x from A, the bending moment is

$$\frac{W}{2}x - W\left(x - \frac{L}{2}\right) = \frac{W}{2}(L - x) = \text{moment of } R_B \text{ about the point considered.}$$

The maximum bending moment occurs at C and is equal to $WL/4$.

CASE IV—SIMPLY-SUPPORTED BEAM WITH NON-CENTRAL CONCENTRATED LOAD

For the loading shown in Fig. 10.14 (a), we calculate the reactions at the supports as follows :—

Taking moments about A :—

$$R_B \times L - W.a = 0$$

$$\therefore R_B = \frac{W.a}{L}$$

$$R_A + R_B - W = 0$$

$$\therefore R_A = W - R_B = W - \frac{W.a}{L} = \frac{W(L-a)}{L}$$

Inserting these values in the loading diagram the shear force diagram is obtained in the same manner as in Case III, and is shown in Fig. 10.14 (b).

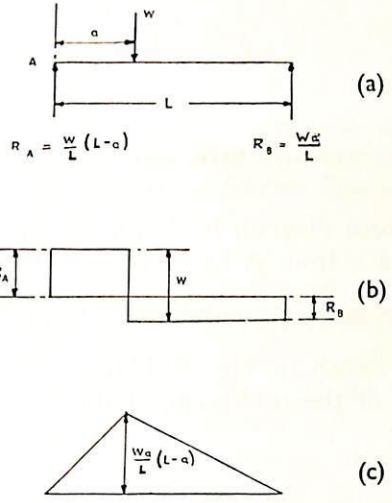


FIG. 10.14.—SIMPLY-SUPPORTED BEAM WITH NON-CENTRAL CONCENTRATED LOAD.

The bending moment diagram, Fig. 10.14 (c), again consists of two straight lines, the maximum bending moment occurring at the point of application of the load and being of value $\frac{W.a(L-a)}{L}$.

CASE V—SIMPLY-SUPPORTED BEAM WITH UNIFORMLY-DISTRIBUTED LOAD (Fig. 10.15 (a))

With a uniformly-distributed load of w per unit length on a beam of length L, the total load is wL, and by symmetry the reactions at the supports are each $wL/2$.

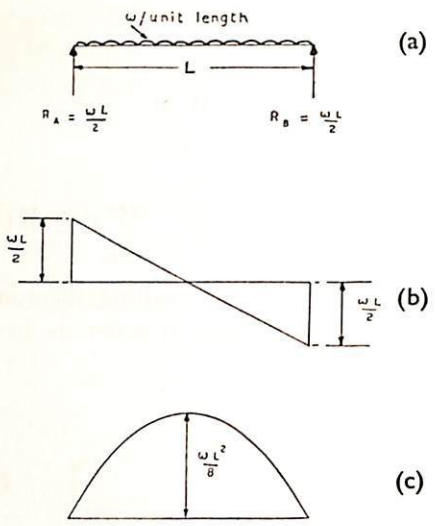


FIG. 10.15.—SIMPLY-SUPPORTED BEAM WITH UNIFORMLY-DISTRIBUTED LOAD.

The shear force diagram is shown in Fig. 10.15 (b), the shear force at distance x from A being $wL/2 - wx$.

The bending moment at distance x from A is $\frac{wL}{2}x - \frac{wx^2}{2}$, and the diagram is shown in Fig. 10.15 (c). The maximum bending moment occurs at the mid-point of the beam, its amount being $wL^2/8$.

CASE VI—SIMPLY-SUPPORTED BEAM WITH UNIFORMLY-DISTRIBUTED LOAD AND OVERHANGING ENDS

The shear force and bending moment diagrams are shown in Figs. 10.16. An alternative way of constructing the bending moment diagram is shown in Figs. 10.17 in which the bending moment diagrams for the loading between the supports and the loading on

the overhanging ends are drawn separately. Although they are of opposite signs, we plot them both on the same side of the base line, the resultant diagram being that contained between the two.

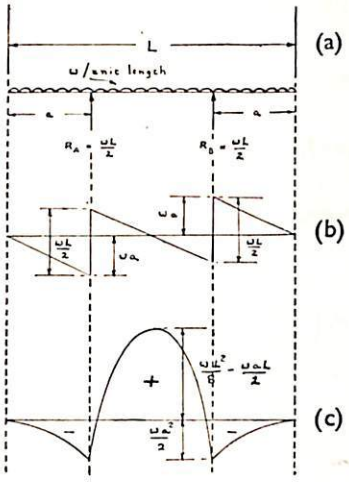


FIG. 10.16.—SIMPLY-SUPPORTED BEAM WITH OVERHANGING ENDS.

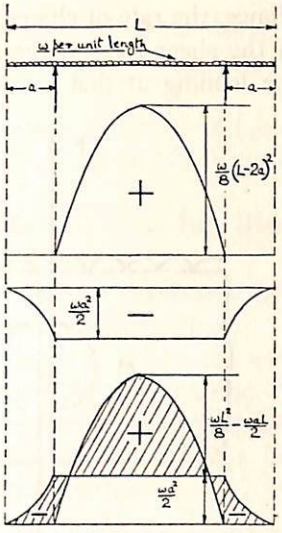


FIG. 10.17.—SIMPLY-SUPPORTED BEAM WITH OVERHANGING ENDS. ALTERNATIVE CONSTRUCTION FOR BENDING MOMENT DIAGRAM.

10.7. Relation between Loading, Shear Force and Bending Moment

Consider an elementary length δx of a beam carrying a continuously distributed load, and let δx be sufficiently small to enable us to assume there is a uniformly distributed load of w per unit length over it. If we isolate this element from the remainder of the beam it will be in equilibrium under the action of the loading and the shear force and bending moment at each end. Let these shear forces be F and $F + \delta F$ and these bending moments be M and $M + \delta M$ as shown in Fig. 10.18.

Since the element is in equilibrium, the algebraic sum of the loads in the vertical direction must be zero.

i.e., $$F + \delta F - F - w\delta x = 0$$

$$\therefore \frac{\delta F}{\delta x} = w$$

As δx tends to zero, this equation becomes

$$\frac{dF}{dx} = w \quad . \quad . \quad . \quad . \quad . \quad \text{Eq. 10.3}$$

Hence, the rate of change of shear force along the beam or the slope of the shear force diagram at any point is equal to the intensity of the loading at that point.

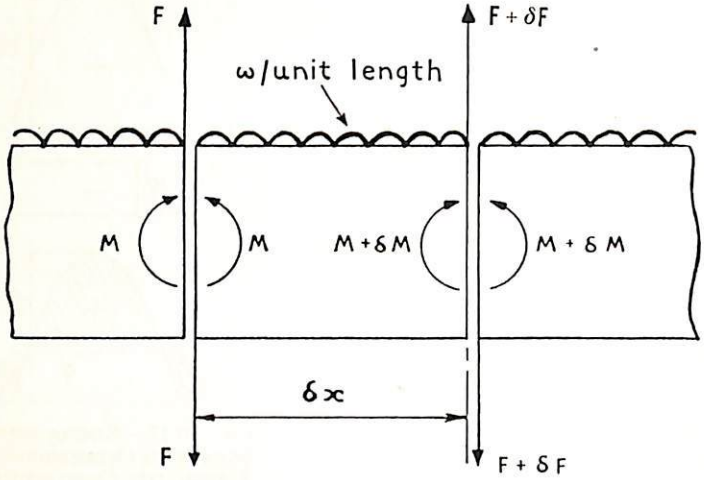


FIG. 10.18.—BEAM ELEMENT.

If now equation 10.3 be integrated between two points x_1 and x_2 at which the shear forces are F_1 and F_2 respectively, we obtain

$$F_2 - F_1 = \int_{x_1}^{x_2} w dx \quad . \quad . \quad . \quad \text{Eq. 10.4}$$

from which it follows that:—The change in shear force between any two points is equal to the area of the loading diagram between them.

Another condition that the forces and moments acting on the element shall be in equilibrium is that the algebraic sum of the moments of all the forces about any point is zero. Therefore, taking moments about any point on the right-hand end of the element,

$$M + \delta M - M - F\delta x - \frac{w(\delta x)^2}{2} = 0$$

i.e.,

$$\frac{\delta M}{\delta x} = F + \frac{w\delta x}{2}$$

As δx tends to zero, this equation becomes

$$\frac{dM}{dx} = F \quad . \quad . \quad . \quad . \quad \text{Eq. 10.5}$$

Hence, *the rate of change of bending moment along the beam, or the slope of the bending moment diagram at any point is equal to the shear force at that point.*

If equation 10.5 be integrated between two points x_1 and x_2 at which the bending moments are M_1 and M_2 respectively, we have

$$M_2 - M_1 = \int_{x_1}^{x_2} F dx \quad . \quad . \quad . \quad \text{Eq. 10.6}$$

whence :—*The change in bending moment between two points on a beam is equal to the area of the shear force diagram between them.*

Application of the above relations enables us to obtain the shear force and bending moment diagram for any type of loading. A convenient tabular method of integrating the loading and shear force diagrams is indicated in the following example, illustrating a case which arises very often in aircraft structural analysis :—

Example. Fig. 10.19 represents the load distribution diagram for a cantilever wing. Draw the shear force and bending moment diagrams.

In the Table which follows, the columns are obtained as under :—

Col. 1 gives distance from the root, x.

Col. 2 gives the ordinate of the loading diagram at x from the root.

Col. 3 is the area of the loading diagram between the point considered and the point immediately before it ; i.e., at 9 ft. from the root the figure in this column is the area of the loading diagram (in lb.) between 12 ft. and 9 ft. from the root.

Col. 4 is the shear force at x from the root, obtained by adding the items in Col. 3 progressively.

Col. 5 is the area of the shear force diagram (in lb. ft.), obtained as Col. 3 is obtained from the loading diagram.

Col. 6 gives the bending moment at x from the root, obtained by progressive addition of the items in Col. 5.

We have simply applied equations 10.4 and 10.6, making use

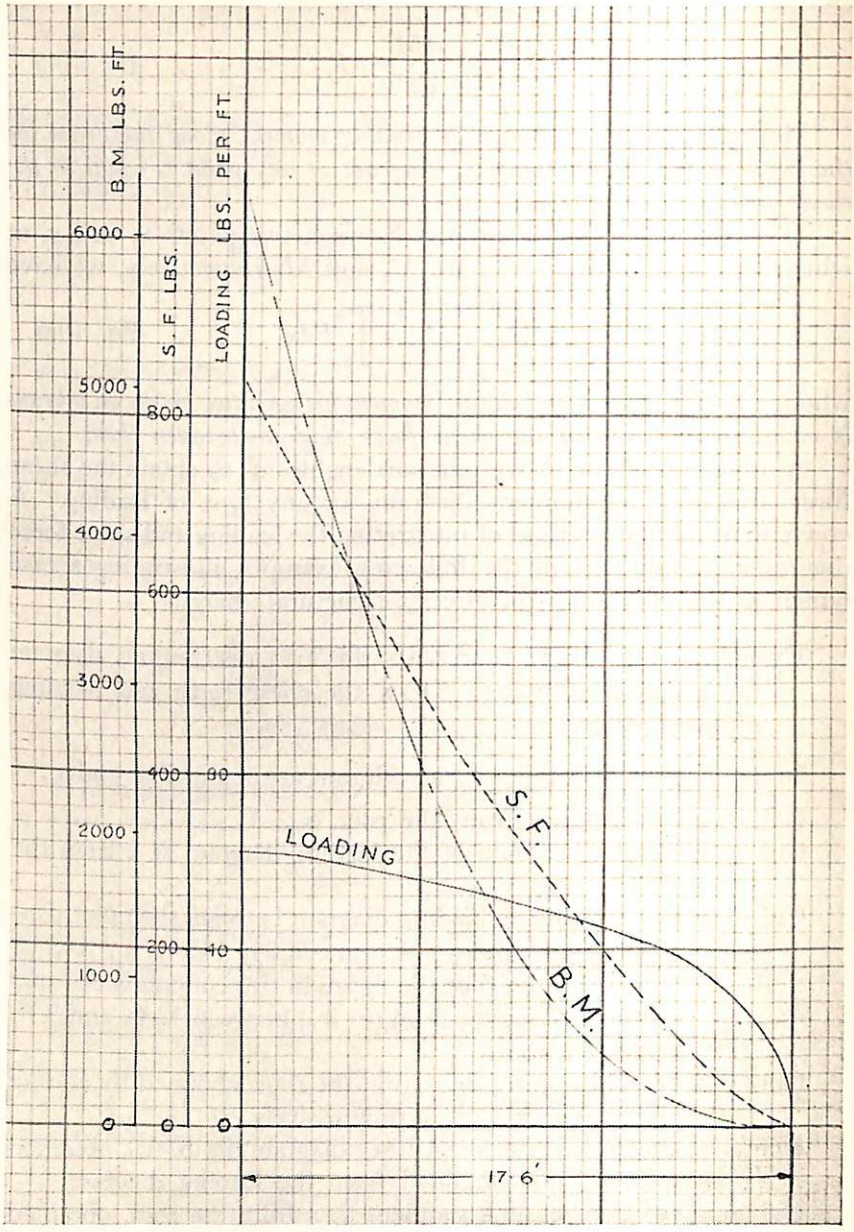


FIG. 10.19.—EXAMPLE. CANTILEVER WITH NON-UNIFORM LOADING.

of the fact that both the shear force and bending moment are zero at the tip of a cantilever.

Dist. from Root x ft.	w lb./ft.	ΔW lb.	S.F. $= \Sigma \Delta W$	ΔM lb. ft.	B.M. $= \Sigma \Delta M$
17·5	0	0	0	0	0
16·5	23·8	12·5	12·5	5	5
15·5	33·4	28·5	41·0	28	33
14·5	37·5	35·0	76·0	61	94
13·5	40·6	39	115·0	94	188
12·0	44·1	68	183·0	262	450
9·0	50·2	143	326·0	775	1225
6·0	55·8	161	487·0	1199	2424
3·0	59·9	172	659·0	1736	4160
0·0	63·0	185	844·0	2106	6366

The areas in the above example were obtained by use of a planimeter, but an approximate value may be obtained by a calculation, in which the diagram is divided up into elemental strips so that the portion of the curve in each strip may be regarded as very nearly a straight line. For instance, the element of the loading diagram between $x = 17{\cdot}5$ and $x = 16{\cdot}5$ is approximately a triangle, the area of which may be obtained as

$\frac{1}{2} \times$ (loading at $x = 16{\cdot}5$) $\times$ base (ft.) $= \frac{1}{2} \times 23{\cdot}8 \times 1{\cdot}0 = 11{\cdot}9$ lb.

Similarly, the element of loading diagram between $x = 16{\cdot}5$ and $x = 15{\cdot}5$ is approximately a trapezium of area,

$$\frac{1}{2} \times (\text{mean loading}) \times \text{base} = \frac{23{\cdot}8 + 33{\cdot}4}{2} \times 1{\cdot}0 = 28{\cdot}6 \text{ lb.}$$

If this method is applied to the above example, it will be seen that the approximation is very close.

Examples, Chapter X

1. A cantilever beam, 8 ft. long, carries concentrated loads of 400 lb., 200 lb. and 100 lb. at distances of 3 ft., 5 ft. and 8 ft. respectively from the root. Draw the shear force and bending moment diagrams.

2. If the 400-lb. load in Example 1 is distributed over the whole length of the beam instead of being concentrated, obtain the shear force and bending moment diagrams.

3. A beam, AB, simply supported at A and B, is 12 ft. long and

carries concentrated loads of 1 ton, 3 tons and 2 tons at 2 ft., 7 ft. and 9 ft. respectively from A. Draw the shear force and bending moment diagrams.

4. Draw the shear force and bending moment diagrams for a beam ABCD, which is 15 ft. long, simply supported at B and at C, distant 3 ft. and 11 ft. respectively from A, if there is a concentrated load of 200 lb. at A, together with a load of 100 lb. at D and a uniformly distributed load of 50 lb. per ft. extending over a length of 6 ft. from A.

5. A cantilever of length 15 ft. carries a distributed load which varies uniformly from 20 lb. per ft. at the tip to 200 lb. per ft. at the root. Draw the shear force and bending moment diagrams.

6. Draw the shear force and bending moment diagrams for a cantilever wing with the load distribution diagram shown in the accompanying table :—

Distance from root, in.	240	234	228	222	216	204	192	168
Loading, lb. per in.	0	19	26·5	32·2	37·0	44·7	51·0	60·6
Distance from root, in.		144	120	96	72	48	24	0
Loading, lb. per in.		67·9	73·5	77·8	81·0	83·2	84·5	84·9

CHAPTER XI

STRESSES DUE TO BENDING OF BEAMS—SIMPLE BENDING THEORY

11.1. General

In the cases usually met with in practice, bending is accompanied by shear, and both shear force and bending moment vary along the length of the beam. The exact determination of the stresses arising in such cases is a complex procedure, and it is customary to calculate stresses on the basis of the " simple bending theory," an explanation of which follows. This theory entails certain simplifying assumptions and, provided discretion is used, it can be employed to give a useful indication of the behaviour of a beam under practical conditions of loading.

11.2. Assumptions

For the simple exposition which follows we shall adopt the undermentioned assumptions—other conditions enter into the precise demonstration of the simple bending theory, but we shall not introduce those until a more advanced stage in the course.

(1) The beam is assumed to be loaded by a pure couple at each end, so that the bending moment is constant along the beam and the shear force is everywhere zero.
(2) It is assumed that cross-sections of the beam which are plane before bending remain plane after bending.
(3) The beam is assumed to be of constant cross-section and of the same material throughout.
(4) The modulus of elasticity is assumed to have the same value in compression as in tension.
(5) The material is assumed to obey Hooke's law, and the stress is assumed not to exceed the elastic limit.

11.3. Simple Bending Theory—Distribution of Stress

Consider the beam shown in Fig. 11.1 (a) loaded by two equal and opposite couples of magnitude M as indicated, so that the bending moment is constant and the shear force is zero.

AB and CD represent two transverse plane sections of the beam, close together, and A′B′, C′D′ represent these same sections when the beam is bent as in Fig. 11.1 (b).

It is apparent that on the upper surface of the beam the fibres are shortened, A′C′ being shorter than AC, and on the lower surface

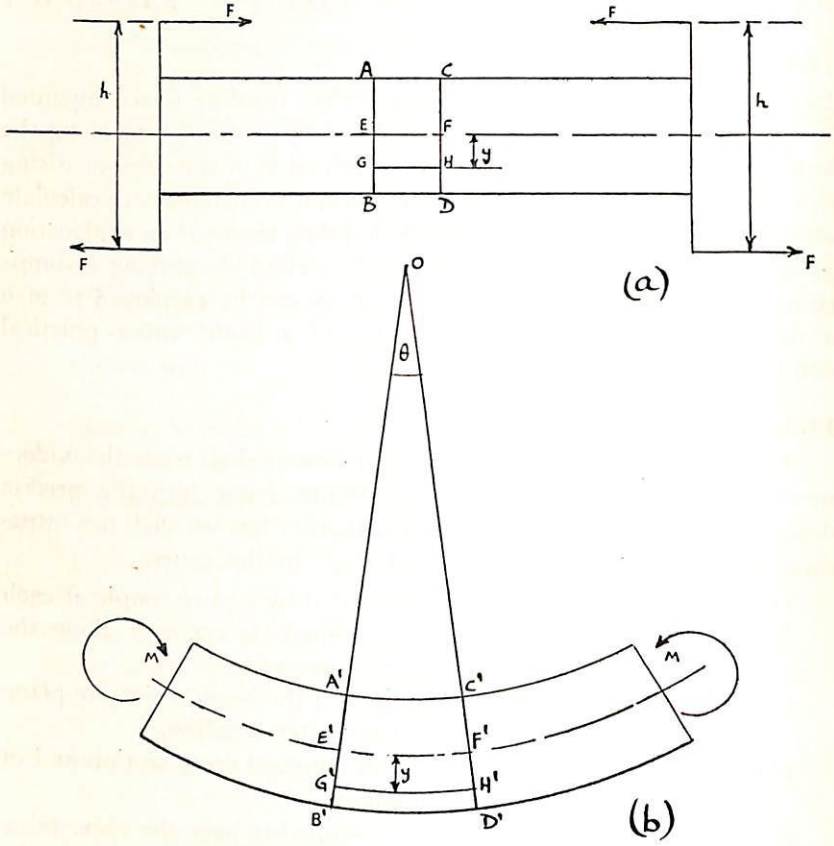


FIG. 11.1.—SIMPLE BENDING.

the fibres are stretched, B′D′ being longer than BD. Since there is an extension on one side of the beam and a contraction on the other side, and since plane sections, such as AB and CD, remain plane after bending, it is obvious that some fibre intermediate between AC and BD will be unchanged in length.

The horizontal surface whose length remains unchanged when

the beam is bent is indicated in the figure by a dotted line, and it is known as the *neutral surface.*

The line of intersection of this surface with a transverse section such as AB is called the *neutral axis* of the section.

It follows that the length E′F′ is equal to the length EF.

Let B′A′ and D′C′ be produced to meet in O at an angle θ.

Let $R = OE' = OF'$ be the radius of curvature of the neutral surface.

If GH is a fibre at distance y from the neutral surface, the change in length of this fibre after bending is equal to

$$G'H' - GH$$

and the strain of this fibre is

$$\frac{G'H' - GH}{GH}$$

But $G'H' = (R + y)\theta$ and $GH = EF = E'F' = R\theta$

$$\therefore \text{ Strain at } y \text{ from neutral surface} \equiv \varepsilon = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R} \quad \text{Eq. 11.1}$$

i.e., *the longitudinal strain at any point in a transverse section is proportional to the distance of that point from the neutral axis.*

Between A′C′ and E′F′ the strain will be compressive; between E′F′ and B′D′ it will be tensile.

If, now, we assume that the modulus of elasticity be the same in tension and compression, equal to E, the tensile or compressive stress at distance y from the neutral surface is given by,

$$f = E\varepsilon = \frac{Ey}{R} \quad . \quad . \quad . \quad . \quad \text{Eq. 11.2}$$

Hence, *the longitudinal stress at any point in a transverse section is proportional to the distance of that point from the neutral axis.*

Fig. 11.2 shows a diagrammatic representation of this variation of stress. In this figure MN and PQ represent the maximum compressive stress and maximum tensile stress, respectively. At

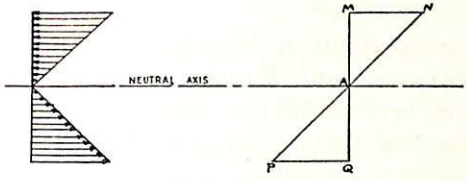


FIG. 11.2.—DISTRIBUTION OF LONGITUDINAL STRESS IN BENDING.

the neutral axis, represented by the point A, the longitudinal stress is zero.

If the modulus of elasticity is not the same in tension and compression, NAP would not be a straight line ; the two portions NA and AP would both be straight but would have different slopes.

However, in what follows we assume E to have the same value in tension and compression.

11.4. Location of the Neutral Axis

Fig. 11.3 (a) represents a transverse section, such as AB, of the beam. Let PQ be a small elemental strip of this section at distance y from the neutral axis, whose area is δA.

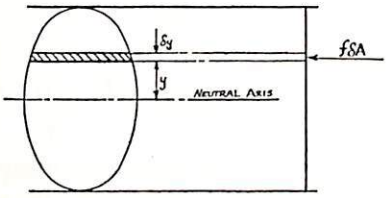


FIG. 11.3.—BEAM CROSS-SECTION.

If the longitudinal stress on this element is f, then the longitudinal load on the element is $f\delta A$, and the total longitudinal load on the section is the sum of the loads on all such elements ; i.e., it is

$$\Sigma f\delta A,$$

the summation extending over the whole section.

Consider now the equilibrium of the section of the beam to one side, say the left, of the section AB which we have represented in Fig. 11.3 (b).

This portion of the beam is in equilibrium under the action of the external couple M and the internal forces acting on the section AB.

Since there is no resultant force acting, but only a couple M, the internal forces acting on AB must constitute a balancing couple. Hence, there is no vertical force on the section, and the sum of the longitudinal forces on the section must be zero.

Therefore, $$\Sigma f\delta A = 0$$

Substituting for f from equation 11.2, this becomes,

$$\Sigma \frac{E}{R} y \delta A = 0$$

But E and R are constant over the section, so that

$$\Sigma y \delta A = 0$$

This means that the total moment of the area of the section about the neutral axis is zero, which can be so only if *the neutral axis passes through the centroid of the section.*

11.5. Value of the Moment of Resistance

We have seen in the previous paragraph that the internal forces acting on the cross-section constitute a couple. The value of this couple is termed the *moment of resistance*, and it is equal to the bending moment.

The moment of the longitudinal force on the element PQ (Fig. 11.3 (b)) about the neutral axis is

$$f \delta A \times y$$

Hence, the total moment about the neutral axis of the longitudinal forces on all the elements of the section is

$$\Sigma f y \delta A = \frac{E}{R} \Sigma y^2 \delta \mathrm{A}$$

This moment must be equal to M for equilibrium.

Therefore, $$M = \frac{E}{R} \Sigma y^2 \delta A \quad . \quad . \quad . \quad . \quad \text{Eq. 11.3}$$

The quantity $\Sigma y^2 \delta A$ has the dimensions of moment of area about N.A. times the distance from N.A., and it is termed the *second moment of area of the section about the neutral axis.* Since it is similar in form to mass moment of inertia, it is often referred to as *moment of inertia*, and is denoted by I.

Using this notation, the equation above may be written

$$M = \frac{EI}{R} \quad . \quad . \quad . \quad . \quad \text{Eq. 11.4}$$

Equations 11.2 and 11.4 may be combined and we have, finally,

$$\frac{M}{I} = \frac{E}{R} = \frac{f}{y} \quad . \quad . \quad . \quad . \quad \text{Eq. 11.5}$$

This equation is of the utmost importance, and should be remembered.

If the value of I is known for the section, the longitudinal stress due to bending at a point distance y from the neutral axis is given by

$$f = \frac{My}{I} \quad . \quad . \quad . \quad . \quad . \quad \text{Eq. 11.6}$$

If y_c and y_t represent the distances of the extreme fibres on the compression and tension sides of the section, respectively, the maximum stresses are given by

$$f_{c_{max.}} = \frac{My_c}{I} = \frac{M}{Z_c}$$

and

$$f_{t_{max.}} = \frac{My_t}{I} = \frac{M}{Z_t}$$

where

$$Z_c = \frac{I}{y_c}$$

and

$$Z_t = \frac{I}{y_t}$$

Z_c and Z_t are known as the *section moduli.*

11.6. Application

Although the equations 11.5 have been derived for a beam of constant section carrying a uniform bending moment with no shear, they can be applied to beams of non-uniform section with varying bending moment accompanied by shear. The results obtained from such application will not be exact, but will give sufficiently close approximation in most cases met with in aircraft structures.

E.g., The maximum bending moment on a beam is 150,000 lb. in. The cross-section has $I = 30$ in.^{4}, and the extreme fibres on the compression and tension sides are distant 3·5 in. and 4·5 in. from the neutral axis, respectively. Determine the maximum compressive and tensile stresses.

Hence,

$$f_{c_{max.}} = \frac{150{,}000 \times 3{\cdot}5}{30} = 17{,}500 \text{ lb. per sq. in.}$$

$$f_{t_{max.}} = \frac{150{,}000 \times 4{\cdot}5}{30} = 22{,}500 \text{ lb. per sq. in.}$$

Determination of I

11.7. The Parallel Axes Theorem

There are two theorems relating to second moment of area which will be found of use in calculating I for various sections. The first of these states that *the second moment of area of a section about any axis in its plane is equal to its second moment of area about a parallel axis through the centroid plus the product of the area and the square of the distance between the two axes.*

This may be proved simply, as follows (Fig. 11.4) :—

Let XX be the axis about which the second moment of area of the section is required. Let this second moment be I_{XX}.

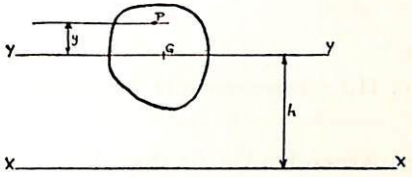


FIG. 11.4.—PARALLEL AXES THEOREM.

YY is an axis through G, the centroid of the section, parallel to XX, and the distance between these two axes is h. The total area of the section is A, and P represents a small element of area, δA, at distance y from the axis YY.

By definition of second moment of area,

$$I_{XX} = \Sigma\, \delta A(y + h)^2 = \Sigma\, \delta A(y^2 + 2hy + h^2)$$
$$= \Sigma\, y^2 \delta A + 2h\, \Sigma\, y \delta A + h^2\, \Sigma\, \delta A$$

The first term is the second moment of area of the section about YY, and we denote this by I_G.

The second term is $2h \times$ the moment of the area of the section about an axis through the centroid and is, therefore, zero.

The third term is $h^2 \times$ the sum of all the elements of area $= h^2 A$.

$$\therefore\ I_{XX} = I_G + Ah^2$$

11.8. The Perpendicular Axes Theorem

The second important theorem states that *the sum of the second moments of area of a section about two perpendicular axes in its plane is equal to the second moment of area about a third axes perpendicular*

to its plane and passing through the point of intersection of the other two axes.

OX and OY are two perpendicular axes in the plane of the section (Fig. 11.5).

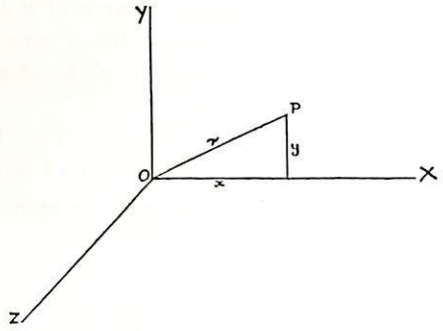


FIG. 11.5.—PERPENDICULAR AXES THEOREM.

OZ is an axis perpendicular to the plane of the section.

P represents an element of area, δA, distant y from OX, x from OY, and r from OZ.

$$I_{OX} = \Sigma y^2 \delta A$$
$$I_{OY} = \Sigma x^2 \delta A$$
$$I_{OX} + I_{OY} = \Sigma y^2 \delta A + \Sigma x^2 \delta A$$
$$= \Sigma (x^2 + y^2) \delta A = \Sigma r^2 \delta A$$

But, $$\Sigma r^2 \delta A = I_{OZ}$$

$$\therefore I_{OX} + I_{OY} = I_{OZ}$$

11.9. Second Moment of Area of Standard Sections

In this paragraph expressions are derived for the second moment of area of certain sections, and the student will find it useful to remember these.

SOLID RECTANGLE (Fig. 11.6)

The C.G. of a solid rectangle is at its mid-point, and for bending in a plane parallel to the sides of length D, the neutral axis will be as indicated by the line N.A.

For the element PQ, of thickness dy, at distance y from the N.A., the second moment of area about the N.A. is $Bdy \times y^2$.

For the whole section,

$$I_{N.A.} = \Sigma By^2dy = \int_{-\frac{D}{2}}^{\frac{D}{2}} By^2dy = B\left[\frac{y^3}{3}\right]_{-\frac{D}{2}}^{\frac{D}{2}} = \frac{BD^3}{12}$$

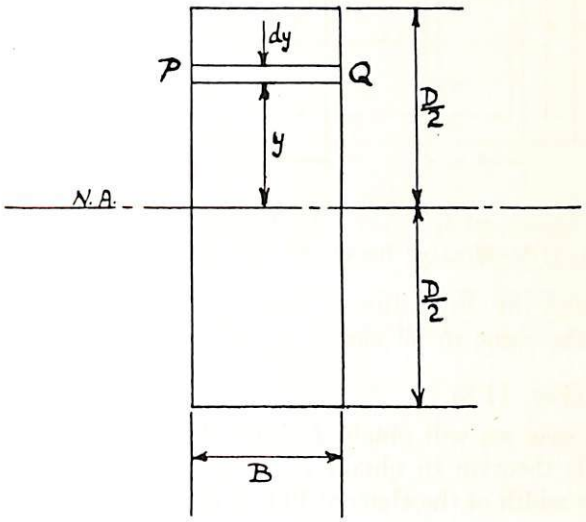


FIG. 11.6.—SECOND MOMENT OF AREA OF RECTANGLE.

It is sometimes convenient to know the second moment of area of a rectangle about its base. Applying the parallel axis theorem, we have,

$$I_{XX} = I_{N.A.} + A\left(\frac{D}{2}\right)^2$$
$$= \frac{BD^3}{12} + BD \times \frac{D^2}{4} = \frac{BD^3}{3}$$

HOLLOW RECTANGLE (Fig. 11.7 (a))

For a hollow rectangle symmetrical about the N.A., we may obtain $I_{N.A.}$ by subtracting the I of the inner rectangle from the I of the outer rectangle.

i.e., $$I_{N.A.} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

Note that this method cannot be applied if the section is not symmetrical about the N.A., since the centroids of both sections would not then lie on the N.A. of the whole section.

It is of interest to observe that the other sections shown in Fig. 11.7 are equivalent to the hollow rectangle for determination

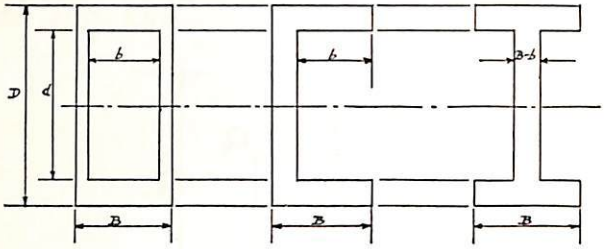


FIG. 11.7.—HOLLOW RECTANGLE AND EQUIVALENT SECTIONS.

of $I_{N.A.}$, since the disposition of material about the N.A. indicated is exactly the same in all cases.

TRIANGLE (Fig. 11.8)

In this case we will obtain I about the base first, and use the parallel axis theorem to obtain $I_{N.A.}$ for the N.A. parallel to the base. The width of the element PQ, at distance y from the base, is

$$\text{PQ} = \frac{D-y}{D} \times B$$

The second moment of area of PQ about $\text{XX} = \left(\frac{D-y}{D} \times B\right) dy \times y^2$

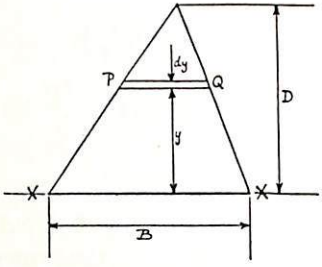


FIG. 11.8.—TRIANGULAR SECTION.

$\therefore$ For the whole triangle,

$$I_{XX} = \int_0^D \frac{B}{D}(D-y)y^2 dy$$

$$= \frac{B}{D}\left[D\frac{y^3}{3} - \frac{y^4}{4}\right]_0^D = \frac{B}{D} \times \frac{D^4}{12} = \frac{\boldsymbol{BD^3}}{\boldsymbol{12}}$$

If $I_{N.A.}$ be the second moment of area about an axis through the centroid parallel to XX, then, since the centroid is $D/3$ from XX,

$$I_{XX} = I_{N.A.} + A\left(\frac{D}{3}\right)^2$$

$$\therefore I_{N.A.} = I_{XX} - A\frac{D^2}{9}$$

$$= \frac{BD^3}{12} - \tfrac{1}{2}BD\frac{D^2}{9} = \frac{\boldsymbol{BD^3}}{\mathbf{36}}$$

CIRCLE (Fig. 11.9)

From symmetry it is obvious that the I of a circular disc about all diameters is the same. In particular, let XOX and YOY be two perpendicular diameters, and let I_0 represent the second moment of area about the axis through O perpendicular to the plane of the disc.

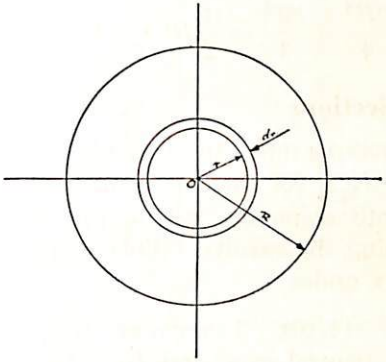


FIG. 11.9.—CIRCULAR SECTION.

Then, $$I_O = I_{XX} + I_{YY} = 2 \times I_{XX}$$

Consider an elementary ring of radius r and thickness dr.

The area of this elementary ring is $2\pi r dr$ and each element of it has its centroid at distance r from O.

Hence, for the elementary ring,

$$I_O = 2\pi r dr \times r^2$$

and for the whole disc, if R be the radius,

$$I_O = \int_0^R 2\pi r^3 dr = 2\pi\frac{R^4}{4} = \frac{\pi R^4}{2}$$

Then, $$I_{XX} = \frac{\pi R^4}{4}$$

If D is the diameter of the disc, we may write

$$I_0 = \frac{\pi R^4}{2} = \frac{\pi D^4}{32}$$

$$I_{XX} = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$$

I_0 is often called the *polar moment of inertia* of the section.

CIRCULAR RING

Outer radius $= R$ Outer diameter $= D$
Inner radius $= r$ Inner diameter $= d$

$$I_0 = \frac{\pi R^4}{2} - \frac{\pi r^4}{2} = \frac{\pi}{2}(R^4 - r^4) = \frac{\pi}{32}(D^4 - d^4)$$

$$I_{XX} = \frac{\pi R^4}{4} - \frac{\pi r^4}{4} = \frac{\pi}{2}(R^4 - r^4) = \frac{\pi}{64}(D^4 - d^4)$$

11.10. Built-up Sections

In aircraft structures much use is made of " built-up " sections, and calculation of $I_{N.A.}$ for these is usually performed by splitting the section up into elements, calculating $I_{N.A.}$ for each of the elements and adding the results. This method is best illustrated by an example, as under.

Example (Fig. 11.10). Determine the cross-sectional area, position of neutral axis, and $I_{N.A.}$ for the section shown.

This type of section is probably best treated by splitting it up into elements as shown in Fig. 11.10 (b). The calculations may be arranged conveniently in tabular form as shown in Table 11.1. The columns in this table are obtained as follows :—

Col. 1 gives the number of the element according to the figure.
Col. 2 gives the area of the element (a sq. in.).
Col. 3 gives the distance of the centroid of the element from the axis XX (y in.).
Col. 4 is Col. 2 multiplied by Col. 3.
Col. 5 is Col. 2 multiplied by (Col. 3)^2 = Col. 4 multiplied by Col. 3.

Col. 6 is second moment of area of element about axis through its centroid parallel to axis XX.

Col. 7 is Col. 5 + Col. 6 = I_{XX} for element.

The sum of the figures in Col. 2 is the area of the section = $\Sigma a = A$.

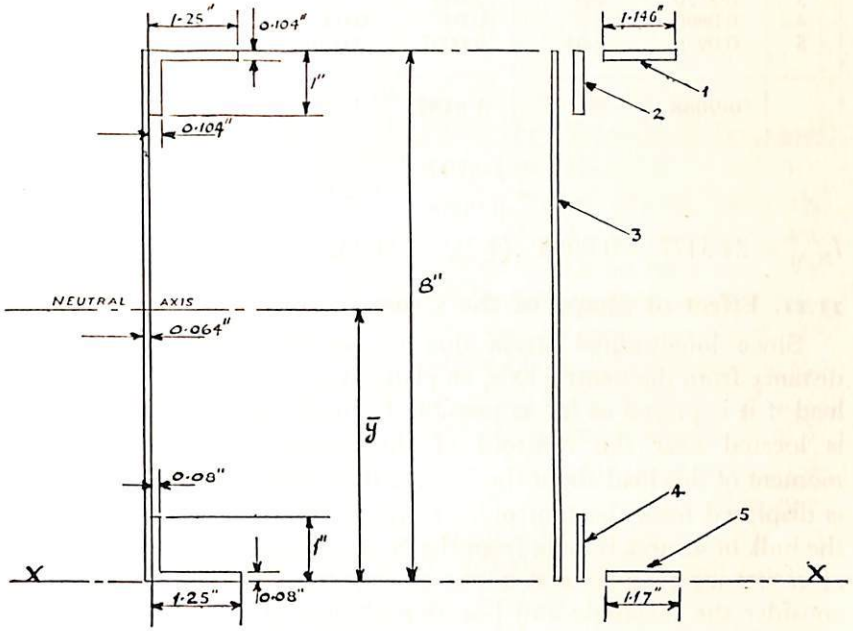


FIG. 11.10.—BUILT-UP SECTION—EXAMPLE.

The sum of the figures in Col. 4 is the moment of the area of the section about XX. Then the distance of the centroid, and hence of the neutral axis, from XX will be given by

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{\Sigma \text{ Col. 4}}{\Sigma \text{ Col. 2}}$$

The sum of the figures in Col. 7 is I_{XX} for the whole section.

Since $I_{XX} = I_{N.A.} + A\bar{y}^2$, it follows that

$$I_{N.A.} = I_{XX} - A\bar{y}^2 = \Sigma \text{ Col. 7} - (\Sigma \text{ Col. 2}) \times y^2$$

TABLE 11.1

Item.	Area = a sq. in.	Distance of Centroid from XX = y in.	ay	ay^2	I_G	$I_G + ay^2$
1	0·1192	7·948	0·948	7·54	0·0001	7·5401
2	0·1040	7·5	0·78	5·85	0·0087	5·8587
3	0·5120	4·0	2·048	8·192	2·73	10·922
4	0·0800	0·5	0·04	0·02	0·0067	0·0267
5	0·0936	0·04	0·0037	0·00015	0·00005	0·0002
	0·9088		**3·8197**			**24·3477**

$$\bar{y} = \frac{3{\cdot}8197}{0{\cdot}9088} = 4{\cdot}2 \text{ in.}$$

$$I_{\text{N.A.}} = 24{\cdot}3477 - 0{\cdot}9088 \times (4{\cdot}2)^2 = 24{\cdot}3477 - 16{\cdot}05 = \mathbf{8{\cdot}2977 \text{ in.}^4}$$

11.11. Effect of Shape of the Cross-section

Since longitudinal stress due to bending is proportional to distance from the neutral axis, an element of area will carry a greater load if it is placed as far as possible from the neutral axis than if it is located near the centroid of the section. Furthermore, the moment of this load about the N.A. will be greater when the element is displaced from the centroid. Consequently, a section which has the bulk of its area remote from the N.A. will have a greater moment of resistance than a section not so constructed. As an example, consider the rectangle and I-section shown in Fig. 11.11, both of which have the same depth and the same cross-sectional area.

Both sections being of the same area,

$$B'D - bd = BD$$

$$\therefore B' = B + \frac{bd}{D}$$

For the rectangle,

$$I_{\text{N.A.}} = \frac{1}{12}BD^3$$

For the I-section,

$$I_{\text{N.A.}} = \frac{B'D^3}{12} - \frac{bd^3}{12} = \frac{1}{12}(BD^3 + bdD^2 - bd^3)$$

$$= \frac{1}{12}\{BD^3 + bd(D^2 - d^2)\}$$

If we consider both sections working to a maximum stress of f lb. per sq. in., then, assuming the dimensions to be in inches, for the rectangle we have

$$M_R = f.\frac{I}{y} = f.\frac{BD^3/12}{D/2} = f.\frac{BD^2}{6}$$

and for the I-section,

$$M_I = f.\frac{I}{y} = f.\frac{1/12[BD^3 + bd(D^2 - d^2)]}{D/2} = f.\left\{\frac{BD^2}{6} + \frac{bd}{6}\frac{(D^2 - d^2)}{D}\right\}$$

which is obviously greater than for the rectangular section.

This simple example serves to show the advantage to be gained by placing most of the material away from the N.A.

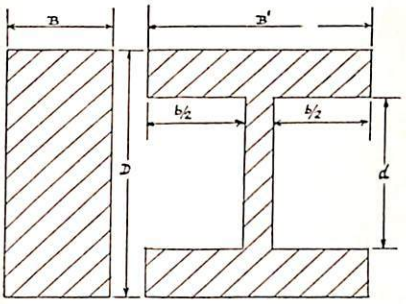


FIG. 11.11.—EFFECT OF SHAPE OF SECTION.

It would appear from the foregoing that it would be most advantageous to have very wide shallow flanges to the I-section, but, as we shall see later, the allowable maximum stress would be much reduced for such flanges owing to the tendency of thin plates to buckle in compression. Failure in this case would be due, not to failure of the material, but to instability of the thin flanges. It must be realised that a limit will be reached beyond which no further advantage is gained on putting more material as far from the N.A. as possible, unless some means is provided of stiffening the thin flanges against buckling. This, however, will be considered in greater detail in some future chapter.

Examples, Chapter XI

1. Find the maximum tensile and compressive stresses in a beam which carries a maximum bending moment of 25,000 lb. in., given the following properties of the cross-section :—$I_{N.A.} = 0{\cdot}5$ in.^{4}; distance of extreme fibre from N.A. = 0·9 in. on compression side and 1·1 in. on tension side.

2. A fin spar is 6 in. deep at the section of maximum bending moment, its cross-section is symmetrical about the N.A. and $I_{N.A.}$

is 5·04 in.⁴. Calculate the maximum bending moment which the spar can withstand if the maximum compressive stress is limited to 10 tons per sq. in. Regarding the spar as a cantilever, 4 ft. long, what load, uniformly distributed over the whole length, would produce this bending moment ?

3. Calculate $I_{N.A.}$ for the section shown in Fig. 11.12. (Note that allowance must be made for bolt holes.) If this section is subjected to a bending moment of 10,000 lb. in., what are the maximum stresses due to bending ?

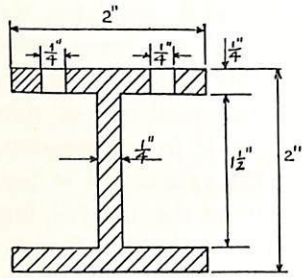


FIG. 11.12.

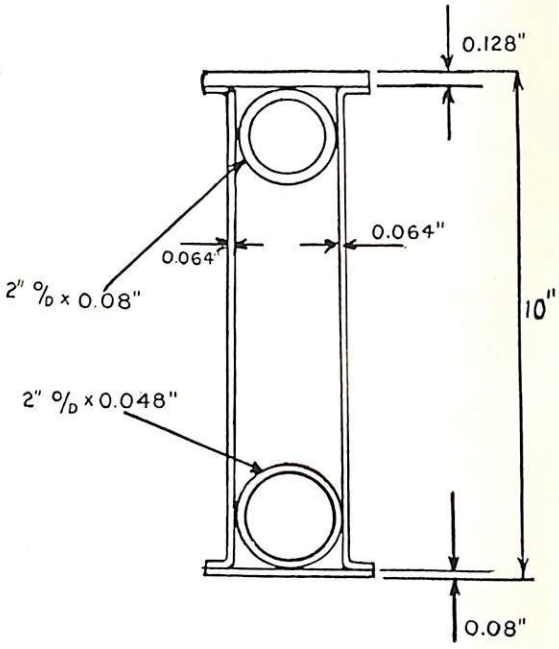


FIG. 11.13.

4. Find the wall thickness required for a tube, 2 in. outside diameter, which has to carry a bending moment of 6750 lb. in., with a maximum allowable compressive stress of 21 tons per sq. in.

5. For the built-up section shown in Fig. 11.13, determine the position of the neutral axis, the cross-sectional area, and $I_{N.A.}$.

6. What uniformly distributed load could the spar whose cross-section is shown in Fig. 11.13 carry, if used as a cantilever of length 9 ft. ? The allowable stress is 15 tons per sq. in.

CHAPTER XII

TORSION

12.1. General

In the preceding chapters we have considered the stresses due to axial loads and due to bending moments or couples applied in planes containing the longitudinal axis. In this chapter we consider the effects of twisting couples applied in planes normal to the longitudinal axis of the member. We shall confine the present discussion to the torsion of circular section members, the theory relating to cross-sections other than circular being too advanced for this stage.

12.2. Torsion of a Circular Section Shaft

Consider the shaft of Fig. 12.1, with one end fixed and a torque T applied to the other end. This torque may be applied by two equal and opposite parallel forces, F, at a distance, d, apart as shown, so that

$$T = Fd$$

We make the following assumptions :—

(1) that plane cross-sections remain plane ; and
(2) that radii such as OB remain straight lines.

Due to the application of torque, the free end will twist relative to the fixed end, the point B moving to B′. A line AB will take up the position AB′.

The angle BOB′, denoted by θ, is the *angle of twist.*

The angle BAB′, denoted by ϕ, is the *shear strain* at the surface.

An angle ϕ may be obtained in a similar manner to give the shear strain at any radius r.

Let the radius of the shaft OB be R.

Then, $$BB' = R\theta. \quad . \quad . \quad . \quad . \quad . \quad \text{Eq. 12.1}$$

For small values of the shear strain, ϕ, we have

$$\frac{BB'}{AB} = \tan\phi = \phi$$

If AB, the length of the shaft, is L, this becomes

$$\frac{R\theta}{L} = \phi \quad . \quad . \quad . \quad . \quad . \quad \text{Eq. 12.2}$$

But, by definition (para. 9.8),

$$\phi = \frac{\text{Shear stress}}{\text{Modulus of rigidity}} = \frac{q}{G}$$

Substituting this value of ϕ in equation 12.2,

$$\frac{R\theta}{L} = \frac{q}{G}$$

i.e.,

$$q = \frac{G\theta}{L} \times R$$

If q be taken to represent the shear stress at any radius, r, by using the value of ϕ corresponding to that radius, we obtain the

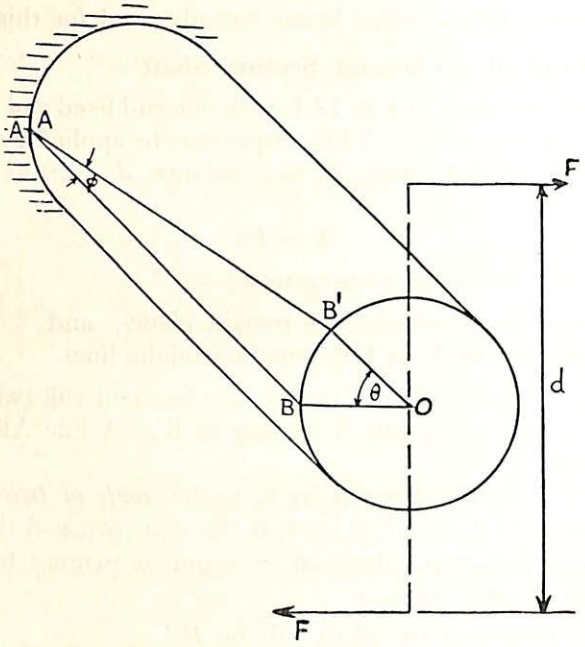


FIG. 12.1.—TORSION OF CIRCULAR SHAFT.

general equation for the shear stress at any point in the cross-section.

$$q = \frac{G\theta}{L} \times r \quad . \quad . \quad . \quad . \quad \text{Eq. 12.3}$$

For any one section θ is a constant, and G and L are constants for the shaft. Hence, the shear stress is proportional to the radius.

Consider now an elementary ring of the cross-section of radius r and thickness δr (Fig. 12.2). The shear stress is constant over this element and equal to q, and will act tangentially—i.e., perpendicular to the radius at every point.

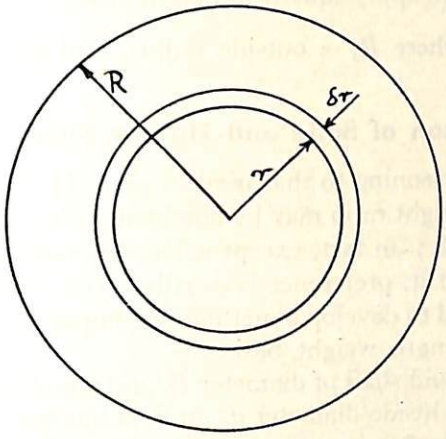


FIG. 12.2.—CROSS-SECTION OF SHAFT.

The area of the element is $(2\pi r) \times \delta r$, so that the shear load in it is

$$q.2\pi r.\delta r.$$

The moment of this force about O $= \delta T =$ force $\times$ radius $= q.2\pi r^2\delta r$.

The sum of the moments of the forces on all such elements of the cross-section must equal the applied torque.

Therefore $$T = \sum_0^R q.2\pi r^2\delta r$$

Substituting for q from equation 12.3,

$$T = \frac{G\theta}{L}2\pi\int_0^R r^3dr = \frac{G\theta}{L}.\frac{\pi R^4}{2} = \frac{G\theta}{L}\mathcal{J} \quad . \quad \text{Eq. 12.4}$$

(where $\mathcal{J} = \dfrac{\pi R^4}{2}$ = polar moment of inertia of the section).

Equations 12.3 and 12.4 may be combined to give us

$$\frac{q}{r} = \frac{G\theta}{L} = \frac{T}{J} \quad . \quad . \quad . \quad . \quad \text{Eq. 12.5}$$

from which, knowing the torque, dimension of the cross-section

(hence $\mathcal{J}$), the value of G for the material and the length of the shaft, we may calculate

(a) the shear stress at any point; and
(b) the angle of twist.

The above will apply equally to hollow shafts, the value of $\mathcal{J}$ being $\frac{\pi}{2}(R_1^4 - R_2^4)$, where R_1 = outside radius, and R_2 = inside radius.

12.3. Comparison of Solid and Hollow Shafts

By similar reasoning to that used in para. 11.11, it is easy to see that a greater weight ratio may be obtained with a hollow shaft than with a solid shaft; in fact, except where limitations on the outside diameter prevent it, preference is usually given to a tubular section. We shall proceed to develop a method of comparing solid and hollow shafts on a strength–weight basis.

Consider a solid shaft of diameter D, and a hollow shaft of outside diameter d_1 and inside diameter d_2, both subjected to torque T, and let the maximum shear stress in each case be the same, equal to f_s.

From equation 12.5, the maximum shear stress occurs at the outside of the section, and we have for the solid section

$$f_s = \frac{TR}{\mathcal{J}} = \frac{T.\frac{D}{2}}{\frac{\pi}{32}D^4} = \frac{16T}{\pi D^3}$$

and for the hollow section

$$f_s = \frac{TR_1}{\mathcal{J}} = \frac{T.\frac{d_1}{2}}{\frac{\pi}{32}(d_1^4 - d_2^4)} = \frac{16T}{\pi\left(d_1^3 - \frac{d_2^4}{d_1}\right)}$$

Since f_s is the same in both cases,

$$D^3 = d_1^3 - \frac{d_2^4}{d_1}$$

i.e.,
$$\left(\frac{D}{d_1}\right)^3 = 1 - \left(\frac{d_2}{d_1}\right)^4$$

or
$$\frac{D}{d_1} = \left[1 - \left(\frac{d_2}{d_1}\right)^4\right]^{\frac{1}{3}} \quad . \quad . \quad \text{Eq. 12.6}$$

If both shafts are of the same material and of the same length, their weights will be in the same ratio as their cross-sectional areas.

$$\text{Therefore,} \quad \frac{\text{Wt. of solid shaft}}{\text{Wt. of hollow shaft}} = \frac{W_s}{W_h} = \frac{\frac{\pi}{4}D^2}{\frac{\pi}{4}(d_1^2 - d_2^2)} = \frac{D^2}{d_1^2 - d_2^2}$$

This may be written

$$\frac{W_s}{W_h} = \frac{\left(\frac{D}{d_1}\right)^2}{1 - \left(\frac{d_2}{d_1}\right)^2} \quad . \quad . \quad . \quad \text{Eq. 12.7}$$

Substituting from equation 12.6 in equation 12.7,

$$\frac{W_s}{W_h} = \frac{\left[1 - \left(\frac{d_2}{d_1}\right)^4\right]^{\frac{2}{3}}}{1 - \left(\frac{d_2}{d_1}\right)^2} = \frac{\left[1 + \left(\frac{d_2}{d_1}\right)^2\right]^{\frac{2}{3}}}{\left[1 - \left(\frac{d_2}{d_1}\right)^2\right]^{\frac{1}{3}}} \quad . \quad \text{Eq. 12.8}$$

It is often convenient to use the ratio of the outside diameter to the thickness in preference to the ratio d_2/d_1.

Since $d_2 = d_1 - 2t$, where t is the thickness of the wall of the hollow shaft,

$$\frac{d_2}{d_1} = 1 - 2\frac{t}{d_1} = 1 - \frac{2}{\frac{d_1}{t}}$$

and by substitution in equation 12.8, the ratio W_s/W_h may be obtained in terms only of the diameter–thickness ratio of the hollow shaft.

For a given hollow shaft, equation 12.8 will give the weight of the solid shaft required to transmit the same torque with the same maximum shear stress.

It is apparent from this equation that the ratio W_s/W_h is greater than 1, so that the solid shaft will be heavier than the hollow shaft.

It must be emphasised that equation 12.8 does not hold in practice for all values of d_1/t, due to the fact that as d_1/t increases the allowable maximum shear stress decreases, since with very thin walled shafts failure would usually occur by buckling or wrinkling of the walls before the ultimate shear stress of the material was reached.

Fig. 12.3 shows W_s/W_h plotted against the diameter–thickness ratio of the hollow shaft using equation 12.8. On the same figure

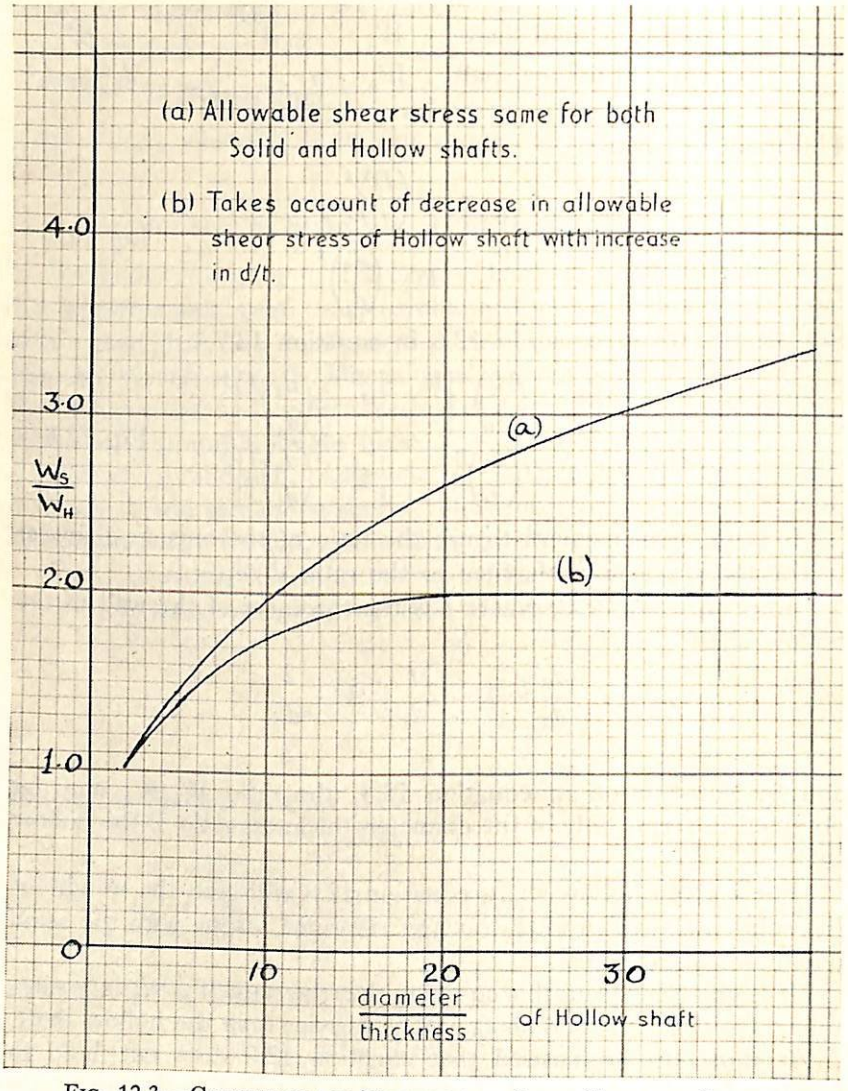


FIG. 12.3.—COMPARISON OF HOLLOW AND SOLID CIRCULAR SHAFTS.

is plotted a curve which allows for the decrease in allowable shear stress with increase in d/t, and which assumes the solid shaft always to work to the ultimate shear stress.

12.4. Transmission of Horsepower by Shafts

The work done by a torque or couple is the magnitude of the torque multiplied by the angle (in radians) through which it turns. Hence, the work done by a torque T in turning through an angle ψ is equal to $T\psi$—provided that T remains constant. If T is varying, the work done is $\int T d\psi$ or equal to the product of the mean torque and ψ.

Horsepower measures the rate of doing work, and 1 H.P. = 33,000 ft. lb. per min. If a mean torque T ft. lb. is applied to a shaft turning at N rev. per min., the torque turns through an angle of $2\pi N$ radians per min. and the work done is

$$T \times 2\pi N \text{ ft. lb. per min.}$$

This is equivalent to a H.P. of

$$\frac{T \times 2\pi N}{33{,}000}$$

Hence, when a shaft is transmitting a horsepower P at N revs. per min., the mean torque on the shaft is given by the relation

$$T = \frac{P \times 33{,}000}{2\pi N} \text{ ft. lb.} \quad . \quad . \quad \text{Eq. 12.9}$$

Example. A steel shaft $2\frac{1}{2}$ in. outside diameter, $\frac{1}{8}$ in. thick, is required to transmit a H.P. of 230 at 2000 revs. per min. What is the maximum shear stress developed and the angle of twist per unit length if the maximum torque is 120% of the mean torque? Take $G = 12 \times 10^6$ lb. per sq. in.

$$\text{Mean torque} = \frac{P \times 33{,}000}{2\pi N} = \frac{230 \times 33{,}000}{2\pi \times 2000} = 604 \text{ ft. lb.}$$

$$= \mathbf{7248 \text{ in. lb.}}$$

$$\text{Max. torque} = 1{\cdot}2 \times 7248 = \mathbf{8698 \text{ in. lb.}}$$

$$\mathcal{J} = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}(2{\cdot}5^4 - 2{\cdot}25^4) = \mathbf{1{\cdot}318 \text{ in.}^4}$$

$$q_{\max.} = \frac{8698 \times 1{\cdot}25}{1{\cdot}318} = \mathbf{8250 \text{ lb. per sq. in.}}$$

$$\theta/\text{unit length} = \frac{8698}{12 \times 10^6 \times 1{\cdot}318} = 0{\cdot}00055 \text{ radians per in.}$$

$$= 0{\cdot}0315° \text{ per in.} = \mathbf{0{\cdot}378 \text{ degrees per ft.}}$$

Examples, Chapter XII

1. A solid steel shaft, 2 in. diameter, is subjected to a torque of 2500 ft. lb. Calculate the maximum shear stress.

2. If the shaft in Example 1 is 18 in. long and if G is 12×10^6 lb. per sq. in., what is the angle of twist?

3. A hollow shaft is 2 in. outside diameter and $\frac{3}{8}$ in. thick. It is observed that an applied torque of 800 lb. ft. produces an angle of twist of 0·21 degrees on a length of 6 in. Calculate the value of G for the material.

4. If the shaft in Example 3 transmits 150 H.P. at 1600 revs. per min., find the maximum shear stress, the maximum torque being 120% of the mean torque.

5. Calculate the torque which will produce a maximum shear stress of 5 tons per sq. in. in a steel bar, 1 in. diameter, and determine the angle of twist on a length of 10 in. if G is 12×10^6 lb. per sq. in.

6. A torque tube in a control circuit is subjected to a torque of 4725 lb. in. The length of the tube is 10 in. and considerations of space limit the outside diameter to 2 in. The material which it is proposed to use is aluminium alloy with an ultimate shear stress of 38,000 lb. per sq. in. and G equal to $3{\cdot}9 \times 10^6$ lb. per sq. in. What wall thickness is required so that the maximum shear stress shall not exceed the ultimate for the material and the angular deflection shall not exceed 5 degrees?

CHAPTER XIII

MATERIALS OF AIRCRAFT CONSTRUCTION

13.1. General

THE materials commonly used in aircraft construction are wood, light alloys, steel and plastics. Wood was the original material used by the early constructors, and has not yet been entirely superseded by metal. In fact, the development of compressed, plastic-impregnated wood has opened up the possibilities of new types of construction.

In a book of this nature it is not possible to discuss aircraft materials in great detail, and it is proposed merely to consider briefly some of the factors governing the choice of materials and the various forms in which they are used. The broad choice between wood and metal has, at times, been decided by the degree of availability of the material. During the 1914–18 war, for example, the scarcity of suitable aircraft timber compelled designers to develop metal structures, which at first followed the same general lines as the wooden, fabric-covered structures then in vogue. The rapid developments in aircraft design during and after that war led to the all-metal, stressed-skin type of structure, which is now the most generally adopted form of construction.

13.2. Light Alloys

(a) *Aluminium Alloys.* Pure aluminium, obtained chiefly from bauxite by chemical and electrolytic processes, is used in sheet form for fairings, cowlings, and other lightly loaded sections, where the fact that it can be welded easily may simplify fabrication. The addition of other elements during the remelting of the pure aluminium provides aluminium alloys which, by virtue of their high strength–weight ratio and ease of fabrication, are the most extensively use metals in modern aircraft construction.

In sheet form, the alloy known as *duralumin* is employed, although more extensive use is now being made of *Alclad* sheet, particularly in cases where good resistance to corrosion is desired.

Alclad is duralumin sheet with a very thin outside coating of pure aluminium, which is bonded to the core by a rolling process.

Extrusions are formed by forcing hot aluminium alloy through the aperture in a die. Just as tooth-paste, squeezed from a tube, assumes the shape of the opening in the top of the tube, so the aluminium alloy takes on the cross-section of the same shape as the aperture through which it is forced. Extruded sections are commonly used for spar booms, stringers, etc.

Forgings and *stampings* of aluminium alloys are also used, particularly for components for aero-engines, such as crankcases and connecting rods. Many of the high-strength fittings in an aeroplane are also often aluminium alloy forgings.

Where very high strength is not important, it is often possible to use a *casting* instead of a forging. Not only is casting generally a less costly process than forging, but also shapes can be obtained which would be impracticable as forgings. High-strength castings can, of course, be produced, but their mechanical properties and resistance to shock loads will be inferior to those of wrought alloys.

(b) *Magnesium Alloys.* Magnesium alloys, having approximately 65% of the weight of aluminium alloys, are the lightest metals used in modern aircraft construction. Their application is, at the moment, limited, owing to their poor corrosion resistance and to the difficulties associated with cold working. They are used chiefly in the form of castings—particularly for small, lightly-loaded components. With future developments in methods of production and fabrication, designers will be able to take increasing advantage of the very low weight of these alloys.

13.3. Steels

Alloy steels of high strength are used quite often, in spite of their high weight compared with aluminium alloys, particularly in cases where a component made from aluminium alloy would be too bulky.

Steel tubing is often adopted as the constructional material for engine mountings and braced fuselages, welding being a commonly used method of forming joints. Stainless steel sheet is gaining increased popularity for such purposes as the covering of wings, as it can be spot-welded easily, and, unless there is a radical change in the methods of wing construction, it may become more generally

adopted as the loads become greater with the future increases in speeds.

13.4. Woods

Wood is still employed in aircraft construction, although, with a few exceptions such as the De Havilland "Mosquito" and "Hornet" and the Kaiser-Hughes flying-boat "Hercules," its use is mainly confined to light aeroplanes and gliders.

Compressed, plastic-impregnated wood has been successfully developed in recent years, and it may well be that a material of this nature, reinforced with a high-strength material such as steel, may form the basis of future methods of construction.

13.5. Comparison of Materials

In choosing a material a designer must have regard to the strength, weight, cost (including cost of fabrication) and durability. He is not concerned merely with using material of high strength, but he must also ensure that the weight of the structure is as low as it can be consistent with the requirements of strength and stiffness. Consequently the strength–weight ratio of two materials should always be considered when comparing them as materials suitable for a certain part of the structure. It must be borne in mind also that a part made from a high-strength material may, if designed purely from a strength point of view, be too flexible, with the consequent necessity of adding more material to provide the requisite stiffness, so adding more weight. It may often be desirable in such a case to use a material of lower strength, in which the part would have sufficient bulk to meet both strength and stiffness requirements without the addition of further material. This means that of two materials having approximately the same strength–weight ratio stiffness requirements will often dictate the use of the material of lower strength in order to avoid the extra weight of the reinforcement which would otherwise be needed. It must, of course, be realised that use of the lower-strength material in such a case, whilst it eliminates the need of additional stiffening, might result in a part too bulky to fit into the space available. Consequently, in the choice of a material, as in many other things, the designer is often forced to compromise. Many developments have taken place in recent years both in the production of materials suitable for aircraft

structures and in methods of fabrication, but the search for the ideal type of construction still proceeds.

13.6. Detail Design

In the succeeding paragraphs and in the accompanying illustrations some examples will be given of design details and of constructional methods. There are, of course, many variations, and it is possible here to give only a very brief survey of typical structural items. The application of the principles covered in the chapters on Strength of Materials to the design and stressing of simple detail parts will be demonstrated by means of numerical examples.

Wing Spars. Many different types of cross-section have been utilised for wing spars, and a representative selection is shown in Fig. 13.1 (wood) and Fig. 13.2 (metal).

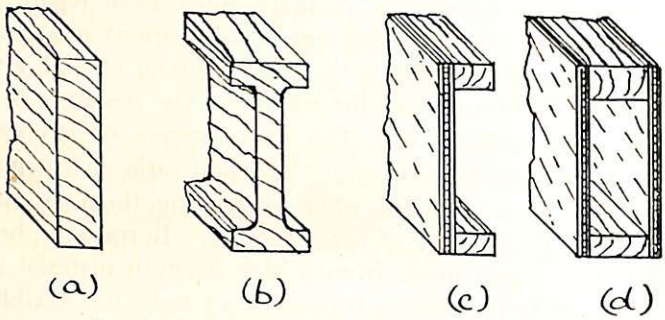


FIG. 13.1.—TYPICAL WOODEN SPAR SECTIONS.

The plain solid rectangular section (Fig. 13.1 (a)) is often found in light aircraft of wooden construction, where ease and cheapness of production are primary considerations. In order to obtain a more efficient spar, it may be spindled to form an I-section as in Fig. 13.1 (b), the increase in efficiency, as indicated by the strength–weight ratio, being apparent on reference to para. 11.11. Other variants use plywood webs and solid flanges, such as the channel section in Fig. 13.1 (c) and the box section in Fig. 13.1 (d).

The metal spars shown are sections commonly used with light alloys as the constructional material, the flanges often being extruded sections. Figs. 13.2 (a) to (d) show sections of the sheet-web type, and Fig. 13.2 (e) represents a typical truss type spar. Stiffeners are necessary on the webs of types (a) to (d) and some of these stiffeners,

often plain angle sections, attached by rivets as indicated in Fig. 13.2(f), may serve as attachment posts for the ribs. It is possible in many

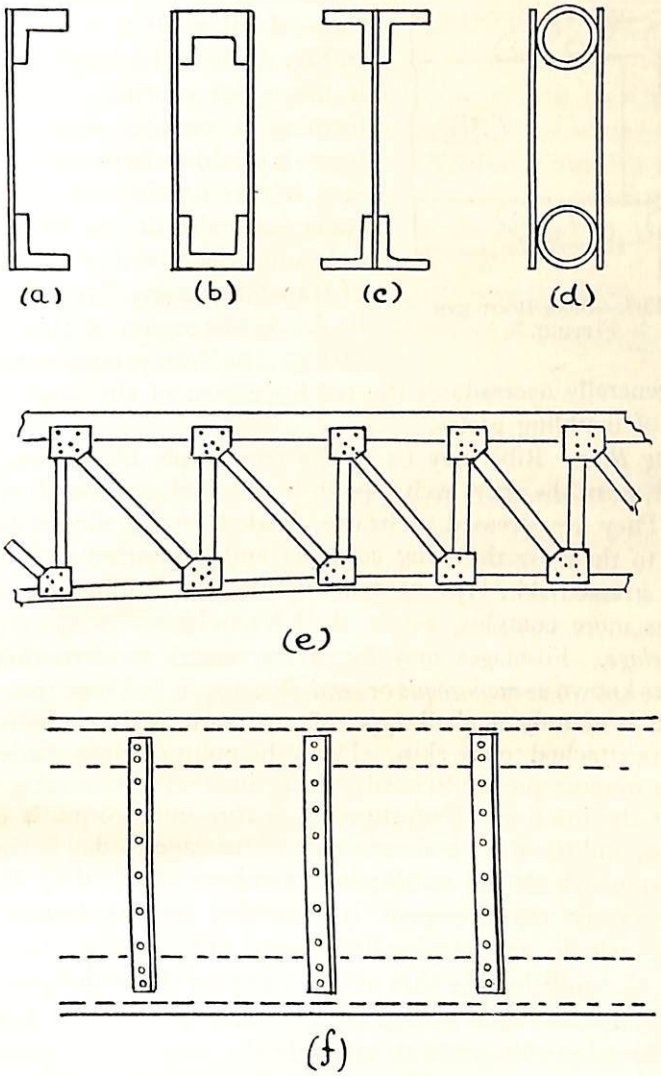


FIG. 13.2.—EXAMPLES OF METAL SPAR CONSTRUCTION.

cases to reduce the weight of the spar and at the same time to provide accessibility and openings, through which control cables or rods may run, by means of flanged lightness holes in the webs.

Spar Attachments. At the root end of the spar the attachment has to be such as to be capable of transmitting a shear force and a bending moment, and the basic form of all such fittings is shown in Fig. 13.3. The bending moment is taken out as two end loads, P, forming a couple, and the shear force is divided between the upper and lower attachment points. In some cases the design is such that only one attachment point is capable of transmitting any "vertical" shear load. In the region of the root-end fittings the heavy concentration of stress generally necessitates the reinforcement of the spar web by means of doubling plates.

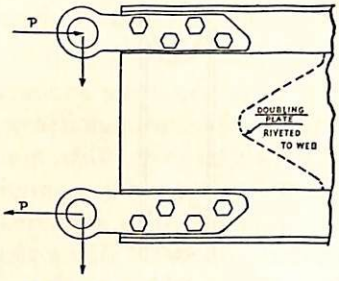


FIG. 13.3.—SPAR ROOT-END FITTING.

Wing Ribs. Ribs may be of the truss type, like those seen in Fig. 7.1, or of the sheet web type in both wood and metal construction. They are stressed as beams, loaded by the air loads transmitted to them by the wing covering and supported at the spars. In the stressed-skin type of construction, the loading on the ribs becomes more complex, as we shall learn later.

Fuselage. Fuselages may be of framework construction or of the types known as *monocoque* or *semi-monocoque.* A true monocoque fuselage is merely a shell type of structure with no reinforcing members attached to the skin. From the point of view of roominess the true monocoque is obviously ideal, since no structural members obstruct the interior. Unfortunately a true monocoque is difficult to realise, and the most common type of fuselage to-day is the semi-monocoque, which has reinforcing members attached to the skin. The necessary reinforcement is provided by (a) *longerons* and *stringers*, which are longitudinal members serving the double purpose of stabilising the skin and of carrying the major part of the end loads due to bending, and (b) *transverse frames* and *bulkheads*, which, like wing ribs, serve to maintain the contour and to stabilise the stringers.

Tail Unit. The general principle of construction of the tail surfaces is the same as that of the wing, and much the same remarks apply.

Stressed-skin Construction. Fabric is not suitable for use as

a covering for the wings of fast modern aeroplanes, and it has given place to plywood or sheet metal. It will be obvious that a covering of wood or metal will contribute to the strength of the structure, and the designer must take account of this if he does not wish to pay the penalty of excess weight.

The effect of introducing a load-carrying skin is to make the wing into a kind of tube. The loads on an aeroplane wing produce torsion as well as shear and bending, and a tubular member is best fitted to carry torsion. In the fabric-covered two-spar wing the torsion is met by " differential bending " of the spars—i.e., it causes one spar to deflect upwards and the other to move downwards. In the stressed-skin wing the torsion is met chiefly by shear in the skin. This produces an additional type of loading on the ribs, since each rib has to transmit to the skin the torque which arises at that particular station.

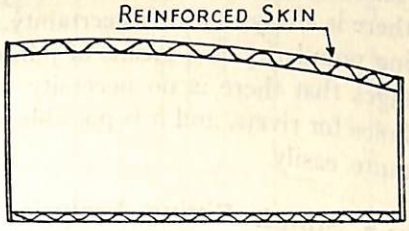


FIG. 13.4.—STRESSED SKIN CONSTRUCTION.

The skin will also assist to some extent in carrying the end loads due to bending, and some designers have built wings so that a considerable portion of the skin can be regarded as spar flanges. In fact, a complete section such as that in Fig. 13.4 may be regarded as a beam section, instead of considering two isolated spars. For this to be done the skin must be either very thick or reinforced in some way, since thin plates cannot withstand large compressive end loads and will buckle at quite low compressive stresses. One means of reinforcing the skin is by the use of corrugations, as in Fig. 13.4, and another, more common, method is to rivet stringers or stiffeners to it. Most stressed-skin structures are designed on the sheet-stringer method, and the study of the behaviour of stiffened thin plates under various types of loading has become of primary importance in aircraft stress analysis. At the present stage in this course we have not advanced sufficiently to enable us to consider this type of construction in detail, and further discussion must be postponed.

Joints. Connections between members in wooden construction are made either by glue or by means of metal fittings bolted to the

members to be joined. Woodscrews are not desirable as they have a tendency to work loose.

In metal aircraft structures, the commonest method of forming joints is by riveting. In cases where the loads are such that the rivets would be subjected to tension, bolts are used instead, since the strength of rivets in tension is very low. Welded joints are found in framed structures, although such joints are less popular in this country than in the U.S.A. One of the major defects of a welded joint is the impracticability of complete inspection; further, the soundness of the joint depends entirely on the skill of the welder. There is no doubt that a welded joint can be exceedingly good, but there is always a little uncertainty. Spot welding is gaining increasing popularity as a means of joining sheet metal; it has the advantages that there is no necessity to weaken the material by drilling holes for rivets, and it is possible to maintain an almost flush surface quite easily.

13.7. Simple Fitting Analysis

The elementary principles of simple fitting analysis will be illustrated by a numerical example. Consider the fitting shown in Fig. 13.5. The procedure could be somewhat as follows:—

Bolt A. This bolt is in double shear, so that we calculate

$$\text{Shear stress} = \frac{3000}{\frac{\pi}{4} \times (\frac{3}{8})^2 \times 2}$$

$$= \mathbf{13{,}575\ lb\ per\ sq.\ in.}$$

Bearing of bolt in fitting

$$\text{Bearing stress} = \frac{3000}{\frac{3}{8} \times \frac{1}{4}}$$

$$= \mathbf{32{,}000\ lb.\ per\ sq.\ in.}$$

Due to the 1-in. offset of the applied load from the face of the web of the fitting, there is a couple of amount 3000 × 1 lb. in., which is assumed to be met by tension on the two bolts at B and by a bearing load on the lower part of the web, and for an approximate determination of the tensile load so produced on the bolts at B we assume that the bearing load can be regarded as equivalent to a load concentrated at C.

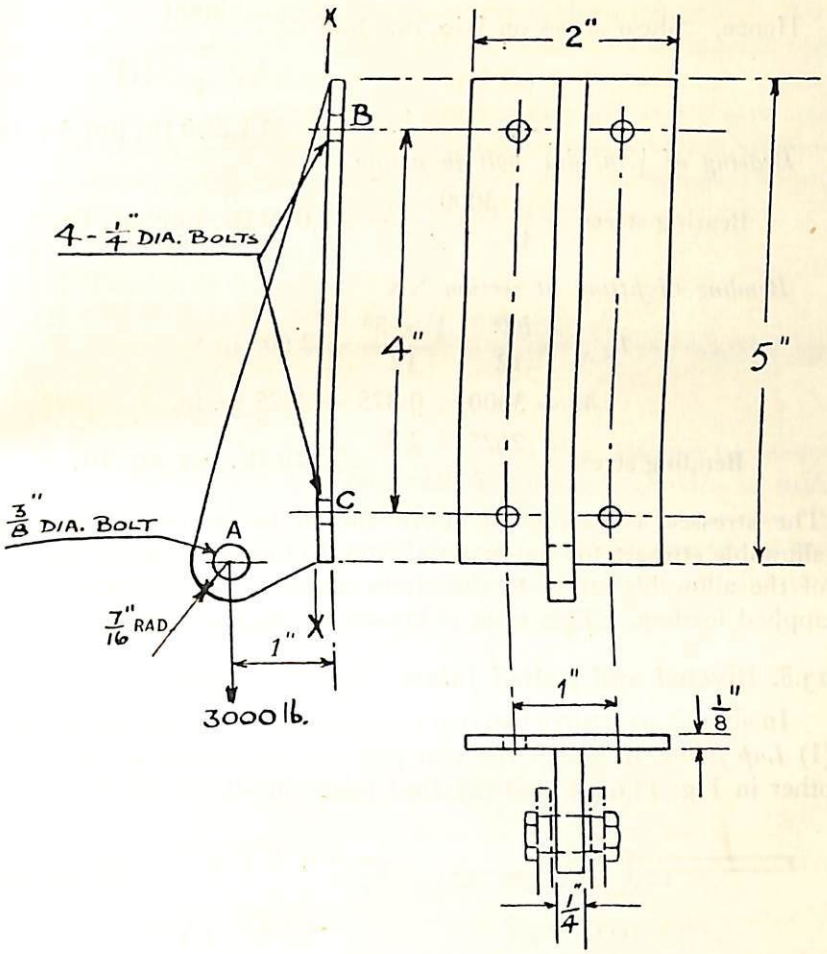


FIG. 13.5.—FITTING ANALYSIS EXAMPLE.

Hence,

$$\text{Tensile load on each bolt at B} = \frac{3000 \times 1}{2 \times 4} = \mathbf{375\ lb.}$$

$$\text{Tensile stress on each bolt at B} = \frac{375}{\frac{\pi}{4} \times (\frac{1}{4})^2} = \mathbf{6690\ lb.\ per\ sq.\ in.}$$

The shear load of 3000 lb. will be shared equally between the four bolts at B and C.

Hence, Shear stress on $\frac{1}{4}$-in. dia. bolt $= \dfrac{3000}{4 \times \dfrac{\pi}{4} \times (\frac{1}{4})^2}$

$= \mathbf{13{,}380\ lb.\ per\ sq.\ in.}$

Bearing of $\frac{1}{4}$-in. dia. bolt in fitting

$$\text{Bearing stress} = \frac{3000}{4 \times \frac{1}{4} \times \frac{1}{8}} = \mathbf{24{,}000\ lb.\ per\ sq.\ in.}$$

Bending of fitting at section XX

$$I_{\text{N.A.}} = \frac{bd^3}{12} = \frac{\frac{1}{4} \times 5^3}{12} = 2{\cdot}605 \text{ in.}^4$$

$$M = 3000 \times 0{\cdot}875 = 2625 \text{ lb. in.}$$

$$\text{Bending stress} = \frac{2625 \times 2{\cdot}5}{2{\cdot}605} = \mathbf{2520\ lb.\ per\ sq.\ in.}$$

The stresses as calculated above should be compared with the allowable stresses for the material, and it is usual to quote the ratio of the allowable stress to the stress which is calculated from the applied loading. This ratio is known as the *reserve factor*.

13.8. Riveted and Bolted Joints

In aircraft structures two main types of riveted joints are used :— (1) *Lap joints*, in which the two plates to be joined overlap each other in Fig. 13.6 (a), and (2) *Butt joints*, in which the two plates

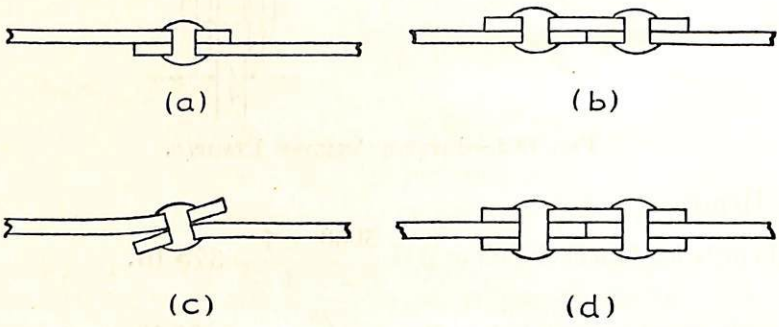


FIG. 13.6.—RIVETED JOINTS.

meet edge to edge, the joining being effected by one or two *butt straps* riveted to each of the plates as in Fig. 13 (b), (d). Sometimes another structural member may serve as a butt strap. With the

lap joint there is a tendency for the plates to distort as in Fig. 13 (c), due to the fact that the applied loads are not in the same straight line. In the case of the lap joint and the butt joint with a single strap the rivets are said to be in *single shear*, and in the butt joint with a strap on each side (double strap) the rivets are in *double shear*.

In checking the strength of a riveted joint consideration must be given to the following possible types of failure :—

(1) Shear of the rivet.
(2) Tension in the plate.
(3) Crushing of the plate or rivet (bearing).
(4) Shear of the plate (rivet bursting through the edge of the plate).
(5) Tension in the rivet.

In general (5) should not arise, as it is bad practice to design a joint so that rivets are in tension, bolts being preferable in such cases. Usually the strength of a joint will be determined by (1) or (3). (2) and (4) will be taken care of by correct spacing of the rivets. In order to avoid (4), it is customary to make the distance

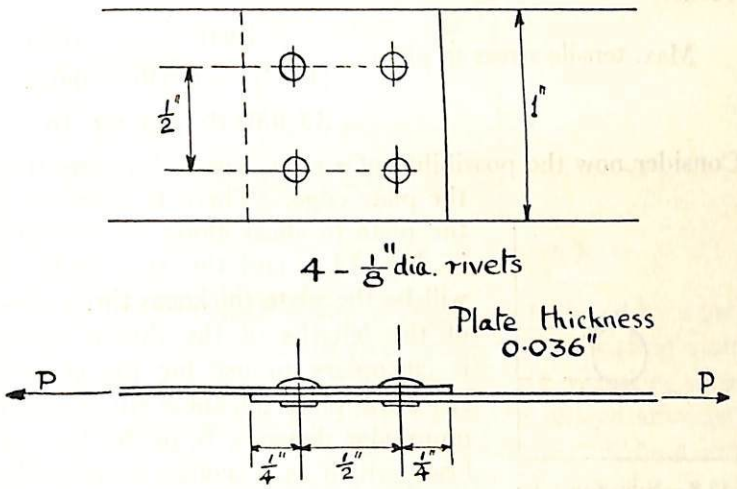


FIG. 13.7.—RIVETED JOINT EXAMPLE.

between the edge of the rivet and the edge of the plate not less than $1\frac{1}{2}$ times the rivet diameter; i.e., the rivet centre should be at least twice the rivet diameter from the edge of the plate.

Consider the simple lap joint shown in Fig. 13.7, where the load P is 1000 lb. The stresses arising would be determined as

follows. In a compact joint it is reasonable to assume that the load is shared equally between the rivets, so that the load on each rivet in this case is 250 lb.

$$\text{The shear stress on each rivet} = \frac{250}{\text{Cross-sectional area}}$$

$$= \frac{250}{\pi/4 \times (\frac{1}{8})^2}$$

$$= \mathbf{20{,}400\ lb.\ per\ sq.\ in.}$$

$$\text{Bearing stress of rivet in plate} = \frac{250}{\text{Rivet dia.} \times \text{plate thickness}}$$

$$= \frac{250}{\frac{1}{8} \times 0{\cdot}036}$$

$$= \mathbf{55{,}500\ lb.\ per\ sq.\ in.}$$

The minimum cross-sectional area of the plate in tension is a section through the centres of two rivets and is equal to (plate width $- 2 \times$ rivet diameter) $\times$ plate thickness.

Hence,

$$\text{Max. tensile stress in plate} = \frac{1000}{(1 - \frac{1}{4}) \times 0{\cdot}036} = \frac{1000}{0{\cdot}027}$$

$$= \mathbf{37{,}050\ lb.\ per\ sq.\ in.}$$

Consider now the possibility of a rivet, say A, bursting through the plate edge. There is a tendency for the plate to shear along the dotted lines in Fig. 13.8, and the area under shear will be the plate thickness times the sum of the lengths of the dotted lines. It is customary to use for the shear area twice the plate thickness times the minimum edge distance, S, in the line of the load, which is a conservative estimate. Some designers use the factor 1·75 instead of 2, thus increasing the safety margin. Using twice the minimum area we obtain,

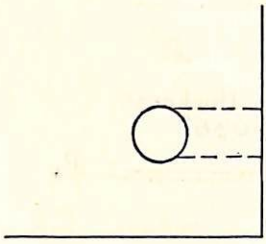


FIG. 13.8.—SHEAR-OUT OR BURSTING.

$$\text{Bursting stress} = \frac{250}{2 \times \frac{3}{16} \times 0{\cdot}036} = \mathbf{18{,}525\ lb.\ per\ sq.\ in.}$$

The stresses obtained by the above calculations should not

exceed the allowable stresses for the material used. An ideally-designed riveted joint would have approximately the same strength in shear of rivets, tension of plate and bearing, but, of course, a compromise is often necessary.

Eccentrically-loaded Joints. So far we have considered only directly-loaded rivet groups—i.e., cases where the applied load can be assumed to pass through the centroid of the group of rivets. However, the type of loading illustrated in Fig. 13.9 is of common occurrence, and we shall now consider how to determine the loads on the rivets in these circumstances.

We assume, first of all, that the plate ABCDE is rigid, and that the whole of any distortion which may arise is due to shear strain

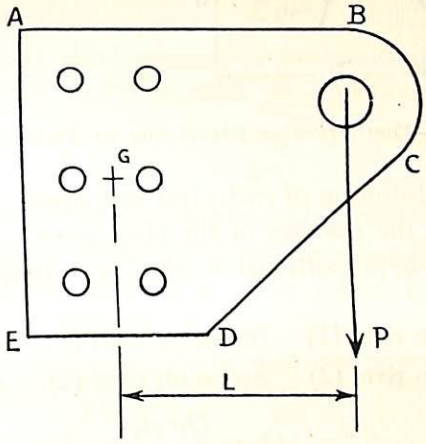


FIG. 13.9.—ECCENTRIC LOADING ON RIVETED JOINT.

of the rivets. The load P acting at O may be replaced by a load P acting at G, the centroid of the rivet group, and a twisting moment about G of value $P \times L$. If the rivets are all critical in shear or all critical in bearing, the load P at G will be shared amongst the rivets in proportion to their areas, so that there will be a load of magnitude $P \times a/\Sigma a$ (where a represents the area of a rivet), on each rivet in the direction of P. If some of the rivets are critical in shear and some in bearing, the rivet value (its strength in shear or bearing as the case may be) is used instead of the rivet area, and G will then be the centroid of the rivet values. The twisting moment will produce a shear force on each rivet at right angles to the line joining the rivet to G, as will be shown below.

Suppose that the plate, which we have assumed rigid, twists about G through an angle θ. If we consider any two rivets as in

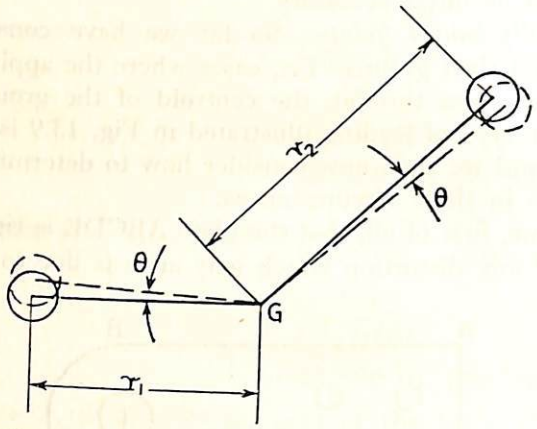


FIG. 13.10.—DISTORTION OF RIVETS DUE TO TWISTING MOMENT.

Fig. 13.10, the distortion of each rivet and hence the strain is proportional to $\theta \times$ the distance of the rivet from G. Furthermore, the shear stress is proportional to the shear strain, and we shall have:

$$\frac{\text{Force on rivet (1)}}{\text{Force on rivet (2)}} = \frac{\text{Stress on rivet (1)}}{\text{Stress on rivet (2)}} = \frac{a_1 \times r_1}{a_2 \times r_2}$$

$$\therefore\ Q_2 = \frac{Q_1 r_2 a_2}{r_1 a_1}$$

where Q represents the force on a rivet due to the twisting moment. This force is obviously at right angles to the line joining the rivet to G, since shear takes place in this direction.

Considering the other rivets similarly, we find

$$Q_3 = \frac{Q_1 a_3 r_3}{a_1 r_1}$$

$$Q_4 = \frac{Q_1 a_4 r_4}{a_1 r_1} \text{ etc.}$$

For equilibrium, we must have that the sum of the moments of these forces, Q_1, Q_2 . . . Q_6 about G, must equal the applied twisting moment $P \times L$.

Hence, $$PL = Q_1r_1 + Q_2r_2 + \ldots + Q_6r_6$$

$$= Q_1\left[r_1 + \frac{a_2r_2^2}{a_1r_1} + \frac{a_3r_3^2}{a_1r_1} + \ldots + \frac{a_6r_6^2}{a_1r_1}\right]$$

$$= \frac{Q_1}{a_1r_1}[a_1r_1^2 + a_2r_2^2 + \ldots + a_6r_6^2] = \frac{Q_1}{a_1r_1}\Sigma ar^2$$

$$\therefore\ Q_1 = \frac{PL \times a_1r_1}{\Sigma ar^2}$$

Similarly, $$Q_2 = \frac{PL \times a_2r_2}{\Sigma ar^2}$$

and, generally $$Q = \frac{PL \times ar}{\Sigma ar^2}$$

If the rivets are all of the same cross-sectional area, and are all critical in shear (or all in bearing), this equation would become

$$Q = \frac{PL \times r}{\Sigma r}$$

Having obtained Q for each rivet, this load is compounded with the direct load on the rivet by any conventional method. The usual procedure is to combine these forces graphically by the

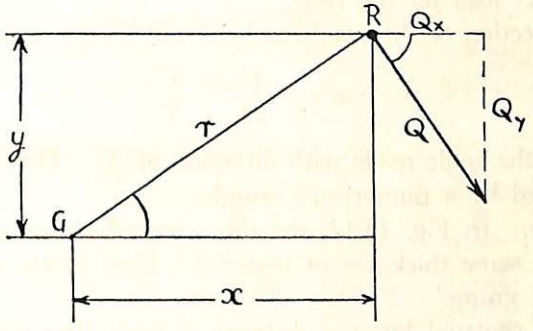


FIG. 13.11.—RESOLUTION OF RIVET LOAD.

parallelogram of forces, but the author has found the following method very useful, involving no graphical construction :—

Consider a rivet, R, and let its distance from the centroid of the rivet group be r. Let x and y be the co-ordinates of R relative to G, measured parallel to two convenient perpendicular axes, and let Q_x

and Q_y be the components of Q in these two directions. Then by similar triangles (Fig. 13.11)

$$\frac{Q_x}{Q} = \frac{y}{r}$$

and

$$\frac{Q_y}{Q} = \frac{x}{r}$$

Hence,

$$Q_x = \frac{Q \times y}{r} = \frac{PL \times ay}{\Sigma ar^2}$$

$$Q_y = \frac{Q \times x}{r} = \frac{PL \times ax}{\Sigma ar^2}$$

Furthermore,

$$r^2 = x^2 + y^2$$

so that

$$Q_x = \frac{PL \times ay}{\Sigma a(x^2 + y^2)}$$

and

$$Q_y = \frac{PL \times ax}{\Sigma a(x^2 + y^2)}$$

If the applied load P is also resolved into the two directions x and y, the composition of the direct load and that due to twisting moment becomes an easy calculation, and may be written as $\sqrt{(Q_x + X)^2 + (Q_y + Y)^2}$, where X and Y are the components of the direct load on the rivet.

The direction of the resultant load will be given by

$$\tan \phi = \frac{Q_y + Y}{Q_x + X}$$

where ϕ is the angle made with direction of X. This method will be illustrated by a numerical example.

Example. In Fig. 13.12, the four rivets have equal areas and bear in the same thickness of material. First locate the centroid of the rivet group.

Let the centroid be at a distance $\bar{x}$ from the line AB and a distance $\bar{y}$ below AD. Then we shall have

$$4\bar{x} = 2 \times 1$$

$$\bar{x} = \tfrac{1}{2} \text{ in.}$$

$$4\bar{y} = 1 \times 1\tfrac{1}{2} + 1 \times 1$$

$$\bar{y} = \tfrac{5}{8} \text{ in.}$$

The force of 500 lb. may be resolved into

500 sin 30° = 250 lb. in the x direction
500 cos 30° = 433 „ „ „ y „

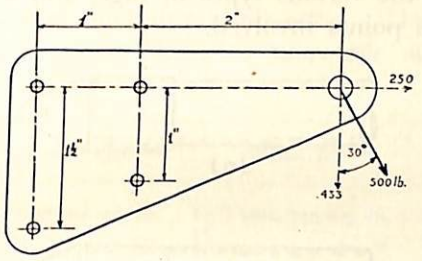


FIG. 13.12.—RIVET GROUP EXAMPLE.

For each rivet $X = \frac{250}{4} = 62{\cdot}5$ lb. →

$$Y = \frac{433}{4} = 108{\cdot}25 \text{ lb.} \downarrow$$

The twisting moment about the centroid is

$$250 \times \tfrac{5}{8} + 433 \times 2\tfrac{1}{2} = 156{\cdot}25 + 1082{\cdot}5$$
$$= 1238{\cdot}75 \text{ lb. in.}$$

The calculation may now proceed in tabular form :—

	Rivet A.	Rivet B.	Rivet C.	Rivet D.	Total.
x in.	0·5	0·5	0·5	0·5	
x^2	0·25	0·25	0·25	0·25	**1·00**
y in.	0·625	0·875	0·375	0·625	
y^2	$\frac{25}{64}$	$\frac{9}{64}$	$\frac{9}{64}$	$\frac{25}{64}$	**1·6875**
Q_x	288→	402←	172←	288→	
Q_y	230 ↑	230 ↑	230 ↓	230 ↓	
$(X + Q_x)^2$	122,850	115,500	12,000	122,850	
$(Y + Q_y)^2$	14,800	14,800	114,200	114,200	
Resultant Force	370	362	355	487	

$$\Sigma r^2 = \Sigma (x^2 + y^2) = 2{\cdot}6875$$

LUGS AND ATTACHMENT OF FITTINGS TO TUBES

In wire-braced structures, the wires are attached to the remainder of the structure by wiring lugs, the connections between the wire

and the lug being by means of a fork end and pin. When tubular members are used in a framework, axial loads are usually applied via a lug, which constitutes part of either a socket or a plug fitting pinned to the tube as in Fig. 13.13. Similar principles are involved in the design of the various types of lugs, and a simple example will illustrate the points involved.

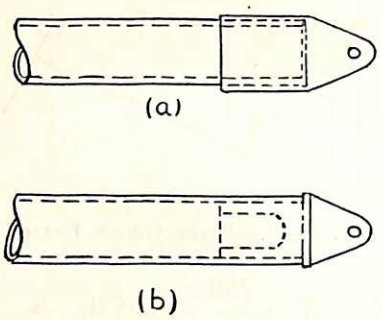


FIG. 13.13.—SOCKET AND PLUG-TYPE END FITTINGS.

A simple plug fitting lug is depicted in Fig. 13.14. It is attached to the tube by means of taper pins, the number of which depends on the shear strength of the pins and the bearing strength of the tube. The wall thickness, t_1, of the plug is determined by the bearing strength of the plug material. The load P is applied to

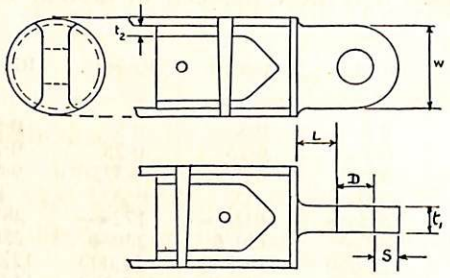


FIG. 13.14.—SINGLE LUG PLUG FITTING EXAMPLE.

the lug by means of a bolt or pin of diameter D, which depends on the shear strength of the bolt or pin. The thickness, t_1, then is determined by the bearing strength of the lug. The dimension S is fixed by considering the bursting strength of the lug, and the width, w, at the bolt hole depends on the tensile strength of the lug. The depth, L, is fixed to give the necessary clearance for the fitting

(usually a fork-end), to which the lug is attached. It may sometimes be necessary to increase t_1, and so reduce S, to provide clearance inside the fork-end.

As an example, let us determine the major dimensions of such a lug which is to be used to transmit a tensile load of 7000 lb. to a $1\frac{1}{2}$-in. dia. 0·028-in. thick duralumin tube. The plug fitting is to be made of dural. The following allowable stresses are to be used :—

	Tensile (f_t).	Shear (f_s).	Bearing (f_{br}).
Tube . .	24 tons per sq. in.	12·5 tons per sq. in.	30 tons per sq. in.
Lug . . .	26 ,, ,, ,, ,,	15 ,, ,, ,, ,,	30 ,, ,, ,, ,,
Taper pins .	—	35 ,, ,, ,, ,,	—
Bolt . . .	—	35 ,, ,, ,, ,,	—

Bolt diameter, D.

If the load is applied from a fork-end fitting, the bolt will be in double shear.

Hence,
$$\frac{7000}{2 \times (\pi/4)D^2} = f_s = 35 \times 2240$$

$$\therefore D = \sqrt{\frac{7000}{2 \times \pi/4 \times 35 \times 2240}} = 0{\cdot}2385 \text{ in.}$$

i.e., **a $\frac{1}{4}$-in. bolt may be used.**

In order that t_1 shall not be excessive, it may be necessary to increase the bolt diameter.

Thickness, t_1.

$$\text{Bearing stress} = \frac{\text{Load}}{\text{Bolt dia.} \times t_1}$$

i.e.,
$$30 \times 2240 = \frac{7000}{\frac{1}{4} \times t_1}$$

$$t_1 = \frac{7000}{\frac{1}{4} \times 30 \times 2240} = \mathbf{0{\cdot}417 \text{ in.}}$$

This thickness is probably excessive, and can be reduced by using a larger bolt, say $\frac{3}{8}$-in. dia. Then

$$t_1 = \frac{0{\cdot}417 \times \frac{1}{4}}{\frac{3}{8}} = \mathbf{0{\cdot}278, \text{ say } \frac{9}{32} \text{ in. } (= 0{\cdot}281 \text{ in.})}$$

This is a reasonable thickness, so use a $\frac{3}{8}$-in. dia. bolt and make $t_1 = \frac{9}{32}$-in.

To determine S, consider bursting or shearing-out of the lug end.

$$\frac{7000}{2 \times \frac{9}{32} \times S} = 15 \times 2240$$

$$S = \frac{7000}{2 \times \frac{9}{32} \times 15 \times 2240} = 0{\cdot}371 \text{ in.}$$

i.e., make $S = \mathbf{0{\cdot}375}$ **in.** $= \frac{3}{8}$ **in.**

Width, w.

Cross-sectional area in tension $= (w - D) \times t_1$

$= (w - \frac{3}{8}) \times \frac{9}{32}$ sq. in.

Allowable tensile stress $= 26$ tons per sq. in.

$$\therefore \frac{7000}{(w - \frac{3}{8}) \times \frac{9}{32}} = 26 \times 2240$$

$$w = \tfrac{3}{8} + 0{\cdot}427$$

$$= \mathbf{0{\cdot}802 \text{ in., say } \tfrac{13}{16} \text{ in.}}$$

This is the minimum width required at the bolt hole.

Making the end of the lug concentric with the bolt hole as in Fig. 13.14, thus simplifying the requirements for S, may give a value for w which is greater than the minimum necessary. In order to

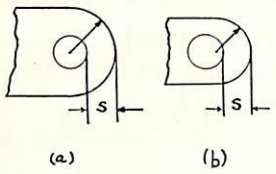


FIG. 13.15.—LUG DESIGN.

avoid the wastage of material thus incurred, the centre from which the end radius is struck is sometimes offset from the centre of the bolt hole as in Fig. 13.15. In this particular example there will, obviously, be more material than is needed between the bolt hole and the tube, but this cannot easily be avoided.

Number of pins required—using $\frac{3}{16}$-in. dia. pins.

Let number of pins be n. Then load on each end of one pin $= \dfrac{7000}{2n}$ lb.

(1) *Shear on pins*

$$\frac{7000}{2n \times \pi/4 \times (\frac{3}{16})^2} = 35 \times 2240$$

$$n = \mathbf{1{\cdot}62—say\ 2\ pins}$$

(2) *Bearing in tube*

$$\frac{7000}{2n \times \frac{3}{16} \times 0{\cdot}048} = 30 \times 2240$$

$$n = \mathbf{5{\cdot}78—say\ 6\ pins.}$$

i.e., 6 pins must be used in order to ensure adequate bearing strength.

Thickness, t_2.

From a strength point of view, this could be the same as the thickness of the tube, since the allowable bearing stress is the same as for the tube, and the number of pins is determined by the bearing strength. From practical considerations this thickness would probably be made larger, say $\frac{1}{16}$-in.

TENSILE STRENGTH OF TUBE

This must be checked at the section carrying the full load of 7000 lb., where the cross-sectional area is reduced by the holes for one taper pin. The cross-sectional area is

$$\frac{\pi}{4}(1{\cdot}5^2 - 1{\cdot}404^2) - 2 \times 0{\cdot}048 \times \frac{3}{16}$$

$$= 0{\cdot}219 - 0{\cdot}018 = \mathbf{0{\cdot}201\ sq.\ in.}$$

$$\text{Tensile stress} = \frac{7000}{0{\cdot}201 \times 2240} = 15{\cdot}55 \text{ tons per sq. in.}$$

The allowable tensile stress is 24 tons per sq. in., so that there is ample reserve of strength.

APPENDIX I

FUNDAMENTALS OF MECHANICS

A.1.1. General

THE student is assumed to be familiar with the fundamental principles of Mechanics, and this Appendix is intended to serve merely as a reminder of those definitions and relationships of which a knowledge is essential for the study of Aeronautical Engineering.

A.1.2. Speed or Velocity

The rate of change of position of a body with respect to time is termed *speed* or *velocity*. If s denotes linear displacement and t denotes time, the velocity, v, at any instant is given by

$$v = \frac{ds}{dt}$$

Typical units are *feet per second* (ft. per sec.), or *miles per hour* (m.p.h.).

Angular Velocity. If O is a fixed point and OA a fixed straight line, and if P is moving in a plane containing OA, the *angular*

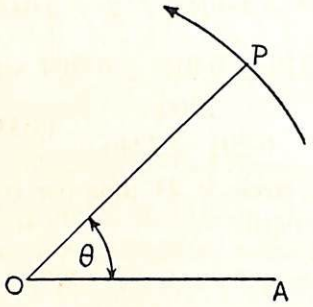


FIG. A.1.1.—ANGULAR VELOCITY.

velocity of P about O is defined as the rate of change of the angle OAP (Fig. A.1.1), and if θ denotes the angle OAP, the angular velocity, ω, is given by

$$\omega = \frac{ds}{dt} \text{ radians per second.}$$

A.1.3. Relative Velocity

The velocity of a point P relative to a point Q may be found by compounding with the velocity of P a velocity equal and opposite to that of Q. Note that velocity, having both magnitude and direction, is a *vector* quantity, and may be represented completely by a straight line, of length proportional to the magnitude of the velocity, drawn in the direction of the velocity.

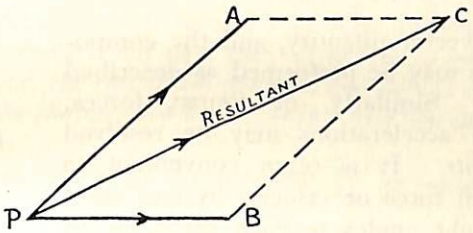


FIG. A.1.2.—COMPOSITION OF VELOCITIES.

Two velocities $\overline{PA}$ and $\overline{PB}$ can be compounded by completing the parallelogram PACB. The *resultant velocity* is then represented by the line PC (Fig. A.1.2).

A.1.4. Acceleration

The rate of change of velocity with respect to time is known as *acceleration*.

$$\text{Linear acceleration} = a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$\text{Angular acceleration} = \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Typical units are :—

Linear, *feet per second per second* (ft. per sec.^{2}).

Angular, *radians per second per second* (rad. per sec.^{2}).

Acceleration, like velocity, is a vector quantity and accelerations can be compounded in the same way as velocities.

A.1.5. Force

Force is generally defined as that which changes or tends to change the state of rest or of uniform motion in a straight line of

a body. If a force P produces on a mass m an acceleration a, then

$$P = kma,$$

where k is some constant. If the system of units is correctly chosen, $k = 1$, and we write

$$\text{Force} = \text{mass} \times \text{acceleration}$$

(See Chapter II for a discussion of the lb.-ft.-sec. system of units.)

Force is a vector quantity, and the composition of forces may be performed as described for velocities. Similarly, of course, forces, velocities and accelerations may be resolved into *components*. It is often convenient to replace a given force or velocity by two components at right angles to each other, as in such a case each component can be considered independently of the other. For instance, the resultant aerodynamic force R on an aerofoil, represented by OC, may be replaced by its components L and D, represented respectively by OB and OA. Then $L = R \sin \theta$, $D = R \cos \theta$ (Fig. A.1.3).

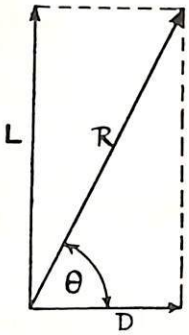


FIG. A.1.3.—RESOLUTION OF A FORCE.

A.1.6. Work and Power

The product of a force and the distance through which its point of application is moved in its line of action is known as the *work done* by the force. Hence, if a force P lb. moves its point of application a distance s in its line of action, the work done is $P \times s$ ft. lb.

The rate at which work is done by a force is known as *power*. At any instant the power is :—

$$P \times \frac{ds}{dt} \text{ ft. lb. per sec.} = P \times \text{velocity}$$

The British unit of power is the *horsepower*, which is equivalent to 550 ft. lb. per sec., or 33,000 ft. lb. per min.

Hence, if a force P moves its point of application in its line of action with a velocity v ft. per sec., the horsepower developed (H.P.) is $\dfrac{P \times v}{550}$.

A.1.7. Momentum

The linear *momentum* of a body of mass m, moving with a velocity v, is defined as the product $m \times v$.

A.1.8. Newton's Laws of Motion

These laws, which form the basis of the theory of Mechanics, may be stated as follows :—

(1) *Every body continues in a state of rest or of uniform motion in a straight line unless acted on by some external force.*

(2) *If a body is not in a state of rest or of uniform motion in a straight line, the rate of change of momentum is equal to the force producing the change, and occurs in the direction in which the force is acting.*

(3) *Every action is accompanied by an equal and opposite reaction.*

The second of these laws justifies our statement in para. A.1.5 that force is equal to mass times acceleration, since acceleration is rate of change of velocity, so that mass times acceleration is the same as mass times rate of change of velocity, which, for constant mass, is rate of change of mass times velocity, i.e., rate of change of momentum.

A.1.9. Energy

The *energy* of a body may be defined as its capacity for doing work. Although energy exists in many forms we are, in this course, concerned primarily with *kinetic energy*, *potential energy* and *pressure energy*, the last of these arising when we consider the energy of an airstream.

Kinetic energy is the energy which a body possesses by virtue of its motion, and for a mass m moving with velocity v is equal to the product $\frac{1}{2}mv^2$.

The change in kinetic energy is equal to the work done by the force which produces the change.

Potential energy is the energy possessed by a body by virtue of its position, and is equal to the work which would be done by the body in moving from its actual position to some standard position. For example, taking the earth's surface as the standard position, the potential energy of a mass m at height h is mgh or Wh, where $W(= mg)$ is the weight of the body.

Pressure energy arises in the case of a fluid or a gas, such as air,

and is measured by the product of the pressure and the volume of the gas concerned.

The principle of the *conservation of energy* states that the total amount of energy in the universe is constant, so that energy can neither be created nor destroyed, although one form of energy may be converted to another. In many of the problems with which we shall be concerned in this course we can neglect the conversion of kinetic and potential energy to other forms (except that we must include pressure energy when we deal with air, and at a more advanced stage heat energy too) and apply the principle in a simplified form, stating that throughout the motion of a body the sum of the kinetic energy and the potential energy remains constant. Bernoulli's equation (para. 3.2) is an application of this principle to the flow of air.

A.1.10. Moment of a Force

The *moment* of a force about any point is an indication of the turning effect of the force and is measured by the product of the

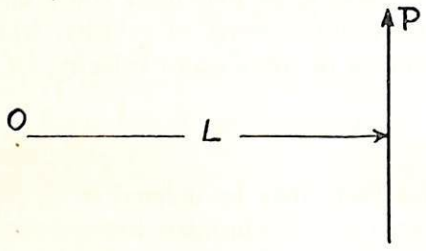


FIG. A.1.4.—MOMENT OF A FORCE.

force and the perpendicular distance from the point to the line of action of the force. In Fig. A.1.4 the moment of the force P about the point O is $P \times L$ lb. in.

Obviously the moment of a force about a point on its line of action is zero.

A.1.11. Couple

If two equal and opposite forces, each of value P, act in the same straight line as in Fig. A.1.5 (a), they will balance each other exactly. If, however, these two forces remain parallel but are no longer in the same straight line, as in Fig. A.1.5 (b), they will not balance exactly. There will be no resultant force, but there will be a turning effect. Such a combination of forces is known as a *couple*, and the

moment of the couple about *any* point in its plane is the product of one force and the perpendicular distance between the forces (in the figure this is $P \times L$).

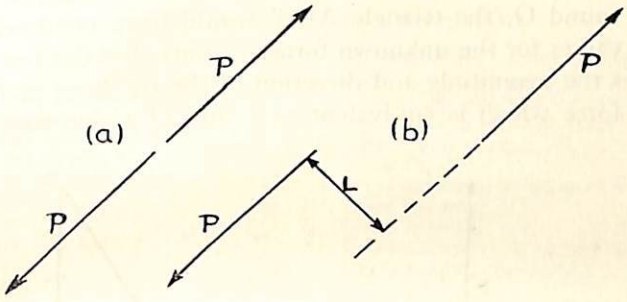


FIG. A.1.5.—DEFINITION OF A COUPLE.

A.1.12. Equilibrium of a System of Forces

If a system of forces is in equilibrium no acceleration will be produced. For a system of coplanar forces the conditions of equilibrium are :—

(1) *The algebraic sum of the resolved parts of the forces in any direction is zero.*

(2) *The algebraic sum of the moments of the forces about any point is zero.*

These two conditions yield three independent equations if we resolve the forces in each of two perpendicular directions. If V and H denote the components of any force of the system in two perpendicular directions (e.g., Vertical and Horizontal), and if M denotes the moment of that force about some chosen point then we may write the conditions of equilibrium as :—

$$\Sigma V = 0\,; \qquad \Sigma H = 0\,; \qquad \Sigma M = 0$$

A.1.13. Triangle of Forces

If three forces acting at a point are in equilibrium, they may be represented in magnitude and direction by the sides of a triangle taken in order round the point. Conversely, if three forces acting at a point can be represented by the sides of a triangle taken in order, they are in equilibrium. For example, if the forces P, Q, R in Fig. A.1.6 (a) are in equilibrium, and if one of them, say P, is known, the magnitudes of the other two may be determined by drawing the triangle

ABC (Fig. A.1.6 (b)), in which the side AB represents P, the side BC represents Q, and the side CA represents R. This triangle has been drawn taking the forces in order in a clockwise direction round the point O. If the forces had been taken in order in an anti-clockwise direction round O, the triangle ABC′ would have resulted, giving the same values for the unknown forces. Note that the line AC (or C′B) gives the magnitude and direction of the *resultant* of P and Q (i.e., the force which is equivalent to P and Q acting together).

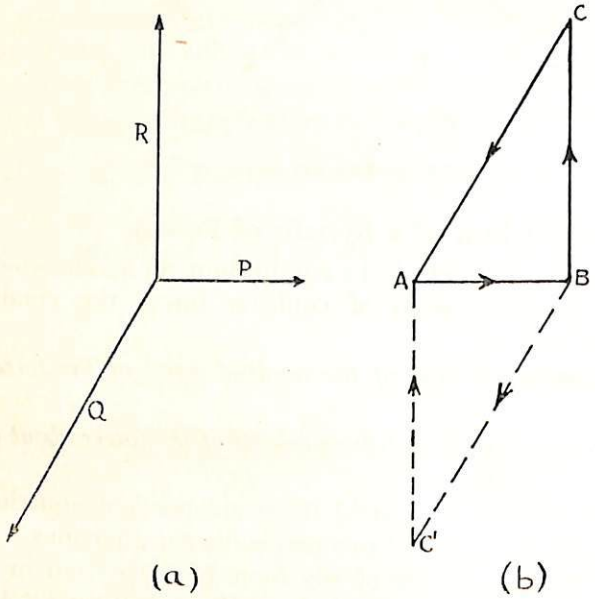


FIG. A.1.6.—TRIANGLE OF FORCES.

A.1.14. Polygon of Forces

If any number of forces acting at a point are in equilibrium they may be represented in magnitude and direction by the sides of a polygon taken in order round the point. The converse is again true.

If a number of forces do not act at a single point but can be represented by the sides of a polygon then the resultant force will be zero, but the forces are not necessarily in equilibrium, since the algebraic sum of their moments about any point will not always vanish in this case.

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